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Directional spectrum of ocean waves from array measurements using phase/time/path difference methods

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Abstract

Wave direction has for the first time been consistently, accurately and unambiguously evaluated from array measurements using the phase/time/path difference (PTPD) methods of Esteva in case of polygonal arrays and Borgman in case of linear arrays. We have used time series measurements of water surface elevation at a 15-gauge polygonal array, in $\cong$ 8 m water depth, operational at the CERC's Field Research Facility at Duck, North Carolina, USA. Two modifications have been made in the methodology. One modification is that we use the *true* phase instead of the *apparent* phase, the other modification being that estimates of wave direction are registered only if the relevant gauges in the array are *coherent* at 0.01 significance level. PTPD methods assume that in a spectral frequency band the waves approach from a single direction, and are simple, expedient and provide redundant estimates of wave direction. Using Esteva's method with the above modifications, we found that at Duck: (i) the directions of swell and surf beat, when energetic swell is present, conform to the schematic diagram of surf beat generation given by Herbers et al., (ii) surf beat of remote origin occurs when the significant wave height, H_{mo}, falls below 0.41 m, (iii) the surf beat of remote origin is not normally incident at the shore contrary to Herbers et al. In fact we found that the surf beat of remote origin is incident at angles in excess of 45° with respect to the shore normal, and (iv) the surf beat of remote origin is largely trans-oceanic in origin. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The traditional approach in dealing with wave direction assumes that the sea surface is composed of a large number of sinusoidal waves of varying frequency and direction, as is exemplified by Pierson et al. (1955) who proposed the frequency–direction energy spectrum:

$$E(f, \alpha) = \phi(f)(2\pi)^{-1} g(\alpha)$$

where $g(\alpha) = \cos^2\alpha, -\pi/2 < \alpha < \pi/2$; $g(\alpha) = 0$ otherwise.

Such unimodal bell-shaped directional spreading function with the peak of the bell generally in the direction of the wind has been observed by Longuet-Higgins et al. (1963) from measurements of a free floating buoy; by Tyler et al. (1974) from synthetic aperture radar observations of radio scatter; and by Donelan et al. (1985) from the measurement of wave elevation at a multi-element array. Since the directional spreading function has been found to be unimodal, the computation of the 'mean' wave direction as a function of frequency is both meaningful and useful and is routinely performed in case of commercially available wave directional buoys fabricated by ENDECO and DATAWELL.

In this paper we present the results of computation of 'mean' wave direction as a function of frequency from linear and polygonal (multi-element) arrays using phase/time/path difference (PTPD) methods, which assume that at a particular frequency band waves can approach from a single direction only—the direction may be different for different frequency bands. In array measurements, wave direction is estimated from the simultaneous time series measurements of water surface elevation at several gauges placed in either linear or polygonal arrays. There are two reasons why PTPD methods for determining wave direction from array measurements have not become popular; the first being the insufficient documentation of Borgman (1974) in case of linear arrays; and the second being the failure of Esteva (1976, 1977) in determining wave direction correctly over the design range 25–7 s of her 5-gauge polygonal array at Pt. Mugu, California.

Borgman (1974) essentially uses the following formula, first given by Munk et al. (1963), for determining wave direction from time series measurements of wave elevation at two gauges:

$$D \sin\alpha = c\tau = c\phi/(2\pi f)\ldots \quad (1)$$

where D is the distance between the gauges; α is the wave direction in the range $(-\pi/2, \pi/2)$ reckoned counter clockwise positive with respect to the seaward normal to the line joining the gauges; f is the frequency; $c(=2\pi f/\kappa)$ is the celerity which is derived as a function of frequency by a process of iteration from the dispersion relation, $(2\pi f)^2 = \kappa g \tanh\kappa d$; $\kappa(=2\pi/\lambda)$ is the wave number and λ is the wave length; d is the water depth; g is the acceleration due to gravity; and τ and ϕ are respectively the time and phase difference between the arrival of a wave crest at the two gauges, the latter being computed as a function of frequency by cross spectrum analysis. Sometimes a small correction, which is evaluated from the coordinates of the gauges, has to be added to the wave direction, α, to make allowance for the two gauges not being exactly parallel to the shore.

The phase difference, ϕ, is the same for wave direction, α, as well as $\pi - \alpha$. Therefore there is an ambiguity within a mirror symmetry in the determination of wave direction by the method of Borgman (1974), i.e. from linear arrays. By this method, just two gauges are sufficient for determining wave direction, i.e. the analysis is done in units of gauge pairs with the different gauge pairs possible in the linear array giving redundant estimates of wave direction.

Esteva (1976, 1977) derived the following formula for unambiguously determining wave direction, α, from simultaneous time series measurements of surface elevation at three non-collinear gauges, i.e. a gauge triad or triangle 123:

$$\propto = \arctan \frac{[(x_1 - x_2)\phi_{13} - (x_1 - x_3)\phi_{12}]\mathrm{sgn}\, p}{[(y_1 - y_3)\phi_{12} - (y_1 - y_2)\phi_{13}]\mathrm{sgn}\, p} \cdots \tag{2}$$

where α is the wave direction in the range $(-\pi, \pi)$ reckoned counter clockwise positive from the positive x-axis; (x_i, y_i), $i = 1, 2, 3$ are the coordinates of the three wave gauges; ϕ_{12} and ϕ_{13} are the phase differences between gauge pairs 12 and 13 respectively and are determined by cross spectrum analysis as a function of frequency; and

$$p = (x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)$$

does not vanish when the three gauges are non-collinear, and by definition $\mathrm{sgn}\, p = 1$ for $p > 0$ and $\mathrm{sgn}\, p = -1$ for $p < 0$.

By the method of Esteva, just three non-collinear gauges are sufficient for determining wave direction unambiguously, i.e. the analysis is done in units of gauge triads, with the different gauge triads possible in the polygonal array giving redundant estimates of wave direction. Esteva used the above formula (Eq. (2)) for obtaining ${}^5C_3 = 10$ redundant estimates of wave direction as a function of frequency from time series measurements of wave elevation at her 5-gauge polygonal array at Pt. Mugu, California, where swell of 16 and 8 s was observed. Esteva using ‘consistency’ within the 10 redundant estimates of wave direction as a criterion for accuracy, reported success in determining direction of 16 s swell, and failure in case of 8 s swell in case of both measured as well as computer simulated data.

Fernandes et al. (1988) provided the necessary documentation in case of linear arrays, and repeating the computer simulations of Esteva for her polygonal array were successful in consistently, accurately and unambiguously determining the direction of both 16 s as well as 8 s swell. Their success was due to their adroit use of the criterion of Barber and Doyle (1956), which stipulates that for ensuring correct directions, the distances between the gauges 12 and 13 in the semi-ordered gauge triad 123, should both be less than half a wave length for the particular frequency band. Thus Fernandes et al. (1988) established that the method of Esteva for determining wave direction unambiguously from polygonal arrays works in case of computer simulated data. The object of the present study is to establish that the method of Esteva works in case of measured data also.

2. Data

The measured wave data used in this study comes from a 15-gauge polygonal array of bottom mounted pressure gauges operational at the Coastal Engineering Research Center's (CERC) Field Research Facility at Duck, North Carolina, USA. The array is comprised of 10 alongshore gauges (operational since March 1987) and five intersecting cross-shore gauges (added in 1990) placed at ≅ 8 m depth (7.44–8.13 m, see Fig. 1). Code names for the 15 wave gauges are as follows: 0 through 9 and A through E. The gauges are placed ≅ 0.5 m above the sea bed.

The data used consisted of 47 three hourly records of time series of wave elevation simultaneously measured at the 15 wave gauges during 1–6 February 1994. Simultaneous time series of pier end wind speed and direction and of barometric pressure were also available. At each gauge the time series record consisted of 5 × 4096 points sampled at 2 Hz, i.e. 0.5 s and was ≅ 2.84 hours long. Starting times of the records were 0100, 0400, 0700, 1000, 1300, 1600, 1900, and 2200 hours of each day. The record for 1000 hours on 1 February was not available. The data was kindly supplied by Dr Charles E. Long of the CERC on two 150 Mb cartridge tapes. Earlier Dr Long had supplied 2 days data (2–3 February 1994) on a 1/2″ magnetic tape along

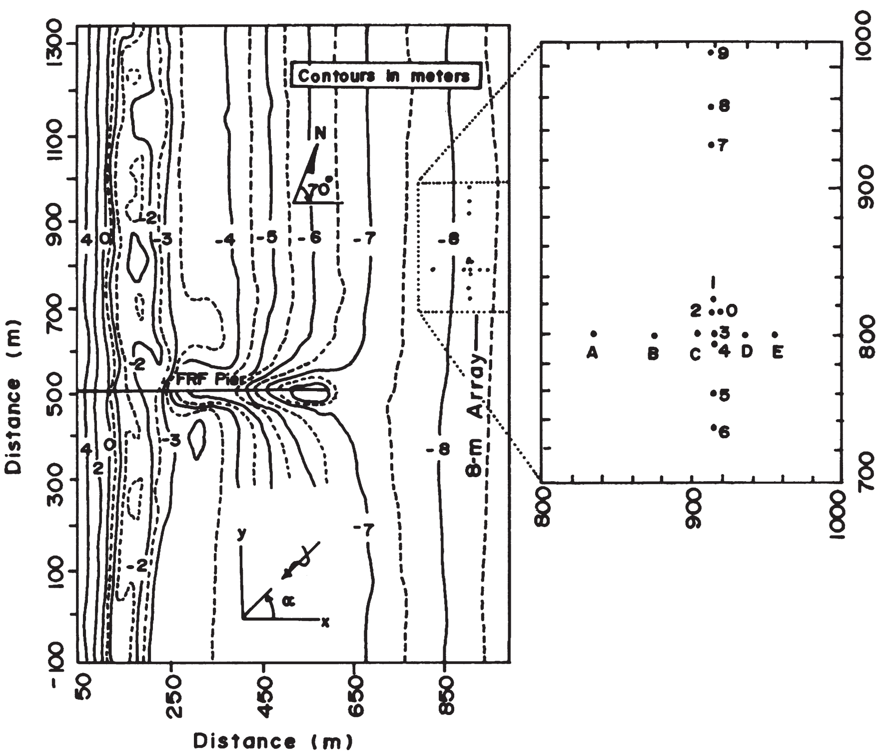

Fig. 1. 15-gauge array at the CERC Field Research Facility at Duck, North Carolina, USA.

with plots of frequency–direction spectra for the same using the Iterative Maximum Likelihood Estimation (IMLE) method. Fernandes et al. (1995) have presented a preliminary analysis of the data on the 1/2″ magnetic tape using PTPD methods.

Long and Roughton (1995) give detailed information about the pressure sensors used, data collection procedure, coastline characteristics, wind and wave climate etc. The coastline at Duck is nearly straight and the depth contours are nearly parallel. The coastline is oriented such that the seaward shore normal makes an angle of 70° with true North. According to Leffler et al. (1989) swell generally comes from the east to the south east; and the most common wave generating winds are Northeasters. The tides are semidiurnal with a mean range of 1 m.

Sources of error in the determination of wave direction are: (a) error in the measurement of the location of the gauges, which at Duck is within $\pm$ 0.15 m; (b) coarseness of the time series sampling interval—this is significant at very high frequencies; and (c) non uniform depth at the gauges in the array and the presence of astronomical tide—at Duck, omitting gauge A, which we have not used, the depth changes by $<$ 0.5 m over the array and the tidal range is small, so that the assumption of a constant mean depth of 8 m at the gauges should not introduce too much error. The errors are diminished substantially, essentially due to the long record length and the large number of gauges available at Duck, which permit a great deal of 'averaging' to be performed. Experiments with computer simulated wave trains, show that the error in the estimation of wave direction at Duck from a single determination (ensemble) is much less than $\pm$ 3° even in presence of random noise.

3. Cross spectrum analysis

Cross spectrum analysis lies at the heart of phase/time/path difference methods. Cross spectrum analysis means the computation of two parameters, viz., phase and coherence between two time series, as a function of frequency (phase spectrum and coherence spectrum). The phase function gives the phase difference (lead/lag) between the two series. The coherence is a measure of the consistency or stability of the relative phase difference between the two series, and like the regression coefficient is defined in the range (0, 1). A coherence of zero indicates that the phase difference has large variance (see Halpern, 1973), so that the average phase difference does not make any physical sense. For performing cross spectrum analysis we have used the Fast Fourier Transform approach as given in Bendat and Piersol (1971). For details please see Fernandes et al. (1995).

The following methodology was followed for computing the cross spectrum analysis of the measured data at Duck:

1. The time series of 5×4096 points sampled at 2 Hz, i.e. 0.5 s, was divided into $M = 80$ ensembles of 256 points each, so that the number of degrees of freedom is 160 and the Nyquist frequency is 1 Hz.
2. Following LeBlanc et al. (1975), Blackman window in time domain was used, so that the half value frequency resolution is 1.64/128 Hz. (Other choices of

windows available in the software were Rectangular, Hanning, Hamming and Cosine Taper). The power spectral densities were scaled up by a factor of 1/0.303 to compensate for loss of energy due to windowing.

3. The power spectral densities were also amplified to account for attenuation of waves with depth using the following formula derived using Linear Wave Theory (see Shore Protection Manual, 1977): $p(\kappa) = \cosh \kappa d/\cosh \kappa b$; where $p(\kappa)$ is the amplitude amplification factor; κ is the wave number; d is the water depth at the gauge site; and b is the height of the gauge above the bottom. To avoid amplification of noise at higher frequencies, the attenuation correction for frequencies above 0.2 Hz was reckoned as equal the attenuation correction for 0.2 Hz (Fig. 2). It may be mentioned that since we perform only ensemble averaging (frequency averaging is not done!), our wave direction estimates are independent of attenuation correction.
4. The coherence squared c^2 as a function of frequency for all frequencies below the Nyquist frequency is computed following Thompson (1979). The threshold values of c^2, which are significant at β significance level are given by; $c^2 = 1 - \beta^{1/(M-1)}$, where M is the number of ensembles. Using this formula it may be seen that for 80 ensembles the threshold value of coherence squared at 0.01 significance level is 0.057, indicating that if the observed coherence squared is larger than 0.057, then in just one case out of 100 the two time series are likely to be not coherent.
5. The phase spectrum ϕ for all frequencies below the Nyquist frequency is uniquely determined in the interval $(-\pi, \pi)$ from the co-spectrum C and the quadrature spectrum Q by taking into consideration of the signs in the numerator and denomi-

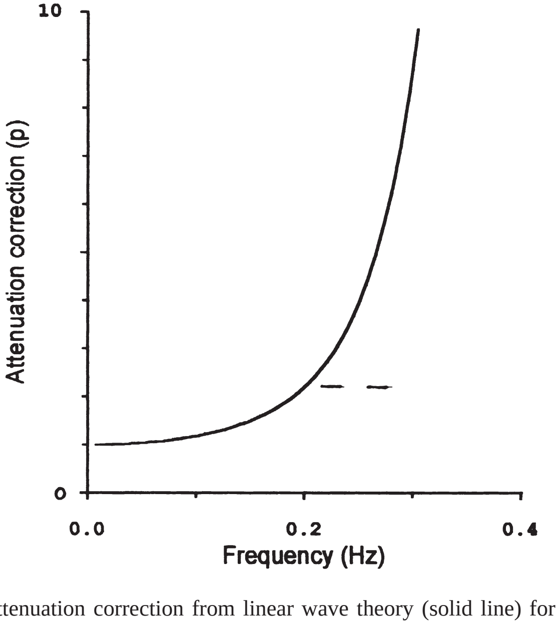

Fig. 2. Attenuation correction from linear wave theory (solid line) for gauge 0.

nator in the formula: $\phi(f) = \arctan[-Q(f), C(f)]$. A positive value of ϕ indicates that the time series x leads over the time series y, i.e., a wave crest first approaches the gauge x and then only the gauge y.

Due to the circular nature of the angular (phase) scale, the above formula gives the ‘apparent’ phase. The ‘true’ phase may be one of the infinite values $\phi + 2j\pi$, where j is an integer (positive, negative or zero). One way of removing this ambiguity in phase is to restrict the ‘true’ phase also to the interval $(-\pi, \pi)$. This can be done by insisting that $|D \sin\alpha| < \lambda/2$. But since α is the unknown parameter to be determined the more stringent condition, $D < \lambda/2$, i.e. the criterion of Barber and Doyle (1956) has to be imposed as has been done by Fernandes et al. (1988) for correctly determining wave direction, in case of computer simulated data.

The above strategy can be used in case of measured data also, but it is wasteful, as it treats perfectly good signal at the higher frequencies as noise and discards it. An inspection of cross spectra from measured data suggests an alternative strategy to determine the ‘true’ phase $\phi + 2j\pi$ from the ‘apparent’ phase ϕ. Generally when the distance between the gauges is $\cong$ 30 m, the gauges are coherent for all frequencies less than about 0.25 Hz and the phase spectrum (‘apparent’ phase) in the above frequency range is a continuous function of frequency evolving from zero phase at zero frequency, except for a few jumps (generally a maximum of two jumps) across the $-\pi/+\pi$ pseudo discontinuity arising due to the circular nature of the angular scale. Such phase spectra may have either positive or negative jumps and the ‘true’ phase is derived from the apparent phase by adding $\pm 2\pi$ for every positive/negative jump (see Fig. 3). The computation of the ‘true’ phase is terminated at the high frequency end at the lowest frequency for which the coherence squared falls below 0.057.

4. Results and discussion

Software in FORTRAN language has been developed to determine wave direction from measurements of surface elevation at a 15-gauge polygonal array at North Carolina, USA, using phase/time/path difference methods of Esteva (1976, 1977) in case of polygonal arrays and Borgman (1974) in case of linear arrays, with two modifications. One modification is that as already stated we use the ‘true’ phase instead of the ‘apparent’ phase; the other modification is that we accept a wave direction estimate as valid only when the relevant gauges are statistically coherent, i.e. the coherence squared > 0.057. The latter modification places an upper limit on the frequency for which wave directions can be correctly computed. Henceforth in this paper when we mention Esteva’s or Borgman’s method, the reference will be to their method as modified by us. Before applying the software to the measured data, the software was applied to computer simulated wave fields at the 15-gauge array and was found to consistently yield directions correctly in case of both linear and polygonal arrays. In the following discussion wave direction indicates the direction *from* which the waves are coming and is measured in the range $(-\pi, \pi)$ counter clockwise positive from the seaward normal to the shore (see Fig. 1).

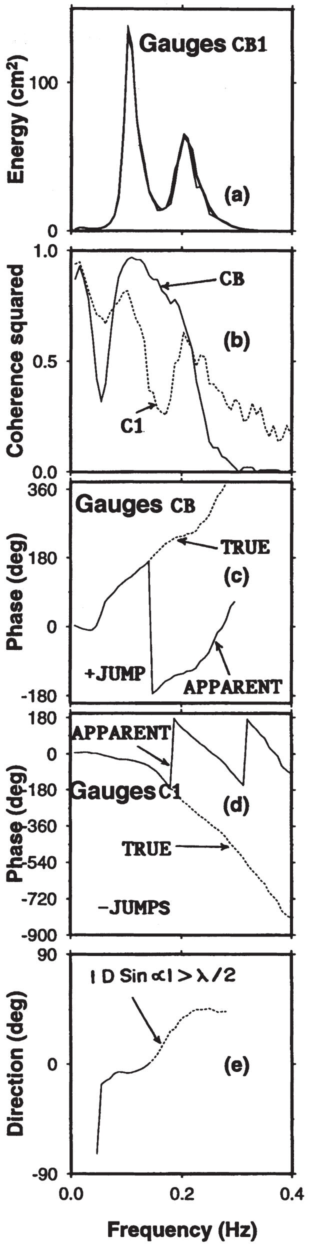

Fig. 3. Analysis of wave directional spectrum using the method of Esteva for the gauge triad CB1 on 2 February 1994 at 0400 hours. Note that estimates of wave direction are available even when the criterion $|D \sin\alpha| < \lambda/2$ is not satisfied (dashed line).

Using the method of Esteva, we have made 15 redundant estimates of wave direction as a function of frequency for each of the 47 records of measured data at North Carolina. The following 15 gauge triads: 012, 34C, D03, D24, C1D, 4C5, D0E, CB1, CB2, CB0, D04, D0C, 02D, 41D and C24; satisfying $D < \lambda/2$ for wave periods above 9 s were chosen as the data showed the presence of swell centred at 9.85 s. Two gauge triads with small gauge separations D, viz., 012 and 34C satisfied the same criterion for periods above 3.7 s. For a gauge triad 123, estimates of wave direction are not available for frequencies $> f_o$, where f_o is the smallest frequency at which the coherence squared for either gauges 12 or 13 falls below the threshold of 0.057. Therefore the number of available redundant estimates of wave direction using Esteva's method is ≤ 15, and this number decreases with increasing frequency. Table 1 gives the number of redundant estimates actually available for the 47 records in case of spectral period 4.27 s. In our discussion hereafter two decimal values in seconds refer to spectral periods. Table 1 shows that in case of 12 of the 47 records, all the 15 redundant estimates of wave direction were available at 4.27 s and that the number of available redundant estimates were ≥ 4. In case of all the 47 records, all the 15 redundant estimates were available for periods ≥ 8.00 s (not shown).

The energy spectra during 1–6 February 1994 show three peaks, for example see the spectra for 2 February 0400 hours (Fig. 3). The peak due to surf beat (i.e. infragravity waves) is generally centred at 42.67 s. The peak due to swell (i.e. waves generated by distant storms) is centred at 9.85 s. The trough between the peaks due to swell and surf beat occurs at 25.60 s. The peak due to sea (i.e. locally generated waves) is prominent during 1 February 2200 hours to 2 February 1300 hours, when it gradually shifts with increasing time from 3.28 to 6.40 s. From the energy contours in Fig. 4(d), it is seen that the swell, which is present throughout the observation period, is dominant (energy $> 10^{2.0}$ cm^2) from the beginning upto 2 February 0400 hours, after which the sea gains dominance, reaching its peak at 1300 hours on the same day (energy $> 10^{2.4}$ cm^2). It is also seen that the sea decays completely by 3

Table 1
Number of available redundant estimates of wave direction at 4.27 s, using Esteva's method. For spectral periods ≥ 4.27 s, the number of available redundant estimates is larger than indicated in this table[a]

		February 1994					
No.	Hours	1	2	3	4	5	6
1	0100	14	5	7	15	8	15
2	0400	13	15	7	15	14	14
3	0700	13	7	7	15	15	11
4	1000	–	6	5	4	15	10
5	1300	12	7	14	13	15	6
6	1600	14	14	15	8	15	8
7	1900	11	8	15	11	9	11
8	2200	8	8	15	7	8	14

[a]N.B. Time series data was not available on 1 February 1994 at 1000 hours.

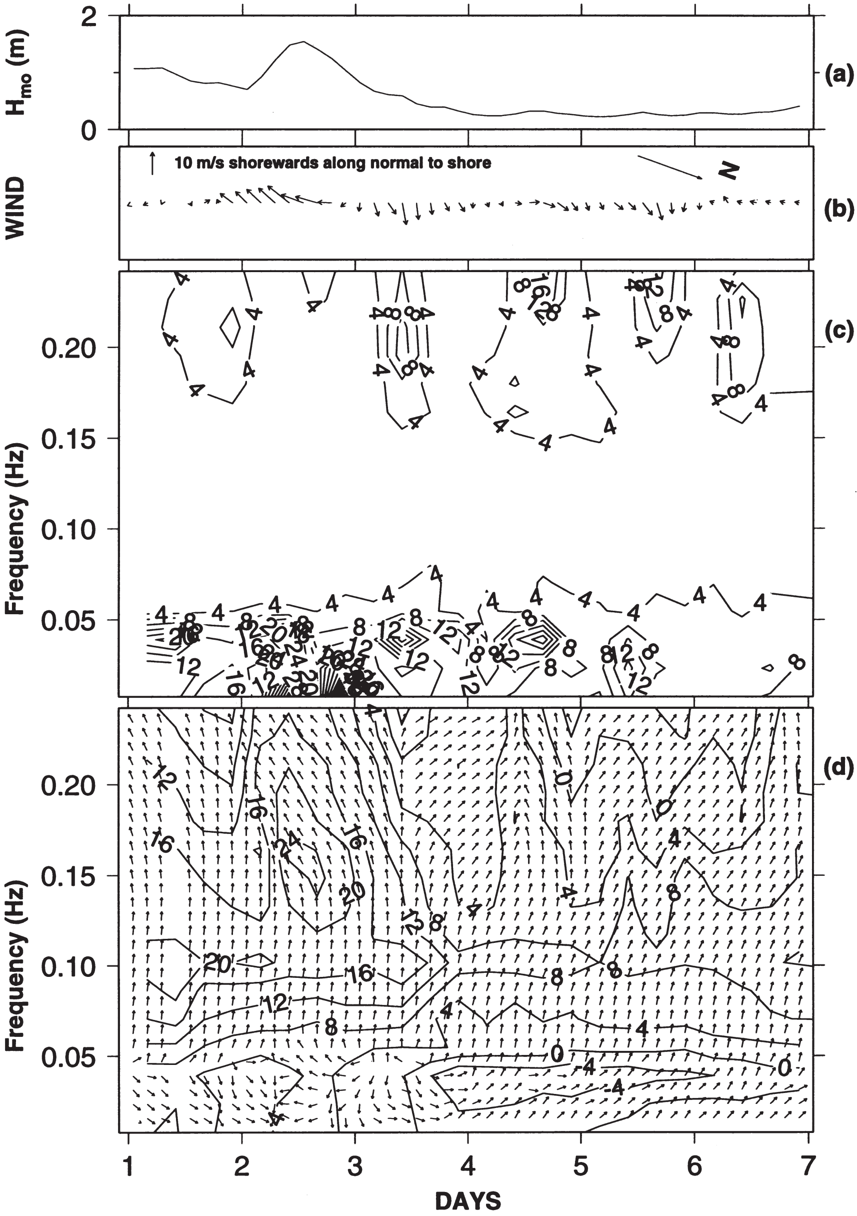

Fig. 4. Wave directional spectrum for the period 1–6 February 1994, computed using the method of Esteva. (a) Significant wave height, H_{mo}, (b) vector mean wind, (c) standard deviation of a maximum of 15 redundant estimates of wave direction, (d) mean of a maximum of 15 redundant estimates of wave direction and energy in decibels ($10 \times \log_{10}$ energy). Energy measured in cm^2. Wave direction is in shore normal coordinate system, just like wind direction.

February 2200 hours after which only very low swell (energy ≅ $10^{0.8}$ cm²) is present, the significant wave height, H_{mo} being less than 0.41 m. The energy values given in braces are for the particular spectral frequencies, i.e. they do not indicate the total energy (variance) for the swell, surf beat and sea components, each of which spreads over several spectral frequencies.

The results of wave direction estimation using Esteva's method have been summarized in Fig. 4. The figure shows that during 1–6 February 1994, a maximum of 15 redundant estimates of wave direction has a standard deviation of $< 4°$ in the range 14.22–6.40 s. During this time swell of ≅ 10 s propagated towards shore, up the coast at an angle of ≅ 7° with the seaward shore normal during the successive dominance of both the swell and the sea, with the angle increasing to and steadying at ≅ 23° during the period of incidence of very low swell on 4, 5 and 6 February. The above standard deviation is also $< 4°$ for periods between 6.40–4.27 s during 0100–1300 hours on 2 February when waves are locally generated (note the energy contour ridge line is oriented in such a way that the frequency decreases with increasing time) by a Northeaster of ≅ 10 m/s, owing to which the wave direction in the period range 6.40–4.27 s aligned itself with the wind direction, and the significant wave height increased from 0.70 to 1.54 m. The low value of the standard deviation indicates that the directional spread in case of sea as well as swell is unimodal and very narrow. In general the standard deviation $< 12°$ in the range 21.33–4.27 s, so that our estimates are reliable. It is curious to note that during 3 and 5 February the wind is blowing *away* from shore, but the high frequency waves generated by this wind are all directed *towards* shore almost at a right angle to the wind. When I brought this apparent anomaly of offshore winds generating onshore waves to the attention of Dr C.E. Long, in a personal communication he informed me that he very frequently sees the phenomenon of wind from the southwest generating high frequency waves from the southeast (our case!), and he sees high frequency waves from the northeast during much rarer occasions when the observed wind is from the northwest. An explanation for this anomaly may be that at Duck the cross-shore component of the wind is fetch limited and the waves generated by the longshore component of the wind are refracted towards shore. The standard deviation of 15 redundant estimates of wave direction is $< 12°$ even for surf beat when low wave conditions ($H_{mo} < 0.41$ m) prevailed on 4, 5 and 6 February indicating that the directional spread is unimodal and narrow. This surf beat (the lowest three rows of vectors in Fig. 4(d) representing 32.00, 42.67 and 64.00 s) propagates up the coast towards the shore at angles in excess of 45° with the shore normal. We note that this surf beat of remote origin does not get refractively straightened as it approaches the coast as much as the swell centred at 9.85 s probably because the 150 m depth contour makes an angle of ≅ 30° with the shore normal. Based on a 24-gauge array at 13 m depth at Duck installed by the Scripps Institution of Oceanography, using a variational method, Herbers et al. (1995) found that surf beat of remote, possibly trans-oceanic origin propagates at normal incidence towards the shore when very low swell energies (variance in the range 0.04–0.14 Hz < 100 cm²) are present. Our results therefore introduce an element of doubt on whether the direction polarization parameter, on the basis of which Herbers et al. (1995) inferred normal incidence, is properly constructed. The standard deviation of 15 redundant estimates of wave direction $< 16°$ also in the case of surf beat when energetic swell was propagating from the beginning upto 2 February 0400 hours, the surf beat propagating up the coast away from the shore at an angle of about 45° with the shore normal. The direction of the surf beat generated by the energetic swell conforms to the schematic diagram of surf beat generation given by Herbers et al. (1995) (their Fig. 2). During the period 2 February 0700 hours to 3 February 0400 hours, the period when locally generated waves are predominant, the standard deviation of 15 redundant estimates of wave direction in case of surf beat is very large, indicating that our assumption of unimodal directional spread is incorrect so that our estimates are not valid in this particular case. It may be speculated that since the swell and the sea both propagate towards the shore, the upcoast and downcoast directions of the swell and the sea respectively give rise to a bimodal directional spread for the surf beat.

We shall now show that, in the wind wave regime (0.04–0.32 Hz), our estimates of wave direction as a function of frequency using Esteva's method, are consistent with the results of frequency–direction spectra obtained by the CERC using the sophisticated Iterative Maximum Likelihood Estimation (IMLE) method described by Pawka (1983), which assumes that at a given frequency waves can approach from all directions. The frequency–direction spectrum for the record on 3 February 1994 at 2200 hours (i.e. when locally generated waves have nearly decayed) using the IMLE method for the full array are shown in Fig. 5(a,b). Superimposed on the IMLE contour plot of energy shown in Fig. 5(b) is a plot (the thick curve down the page!) of wave direction (mean of a maximum of 15 redundant estimates) as a function of frequency using Esteva's method, which shows that the results of IMLE and Esteva's methods are consistent with each other. In Fig. 6, which is similar to Fig. 5 and is for the record on 2 February 1994 at 1300 hours (i.e. at the peak of wave generation) for the full array, we note that the curve showing Esteva's method surprisingly does not exactly follow the ridge on the IMLE contour plot at the high frequency end of the spectrum. Since the wave direction in Fig. 6 varies within the range $(-90°, +90°)$ there will be no ambiguity in the IMLE analysis even if it is restricted to the linear part, i.e. the longshore part of the array—this is given in Fig. 7 which shows that the curve depicting Esteva's method lies exactly along the ridge of the IMLE contour plot even at the high frequency end of the spectrum. Thus the results of Esteva's method are consistent with that of the IMLE method. These methods were found to be consistent not only for just the above two records, but also for all the 16 records on 2 and 3 February 1994 for which the IMLE plots were available, and which as we have seen include instances of predominant swell, a developing sea and a decaying sea. Our estimates of wave direction using Esteva's method (Fig. 4) are also consistent with the CERC IMLE derived bulk parameters, viz., peak period, peak direction etc., given in Fig. 8, which is reproduced from Long and Roughton (1995). We note that our representation explains the singularities in the peak period and peak direction (bulk parameters) as given by the CERC. Also the nearly unchanging direction of swell as shown by our representation appears to be realistic. Fig. 8 shows that $H_{mo} < 0.41$ m during 15 February 1800 hours to 16 February 1200 hours from which we can predict that surf beat of remote origin will

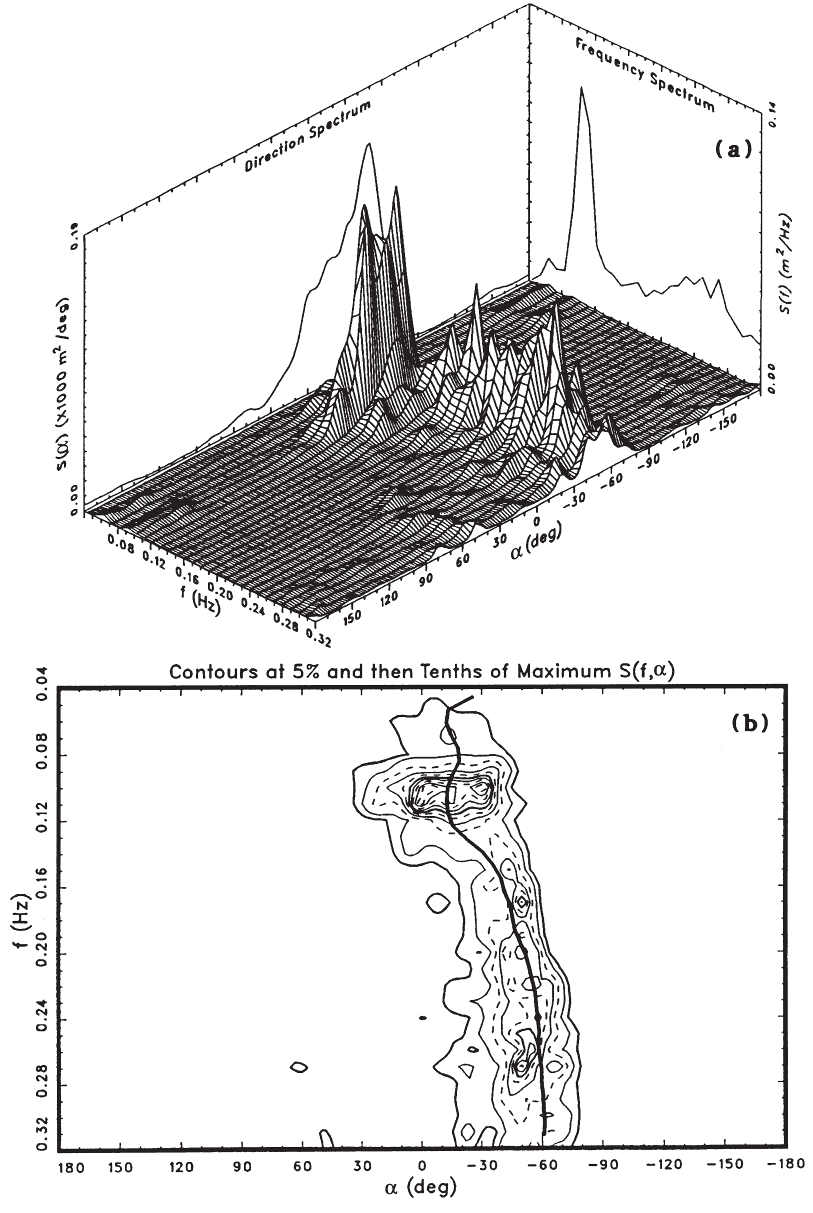

Fig. 5. Frequency–direction spectrum for the record on 3 February 1994 at 2200 hours for the full array using IMLE method. In part (b) the thick curve down the page gives the wave direction (mean of 15 redundant estimates) computed using Esteva's method.

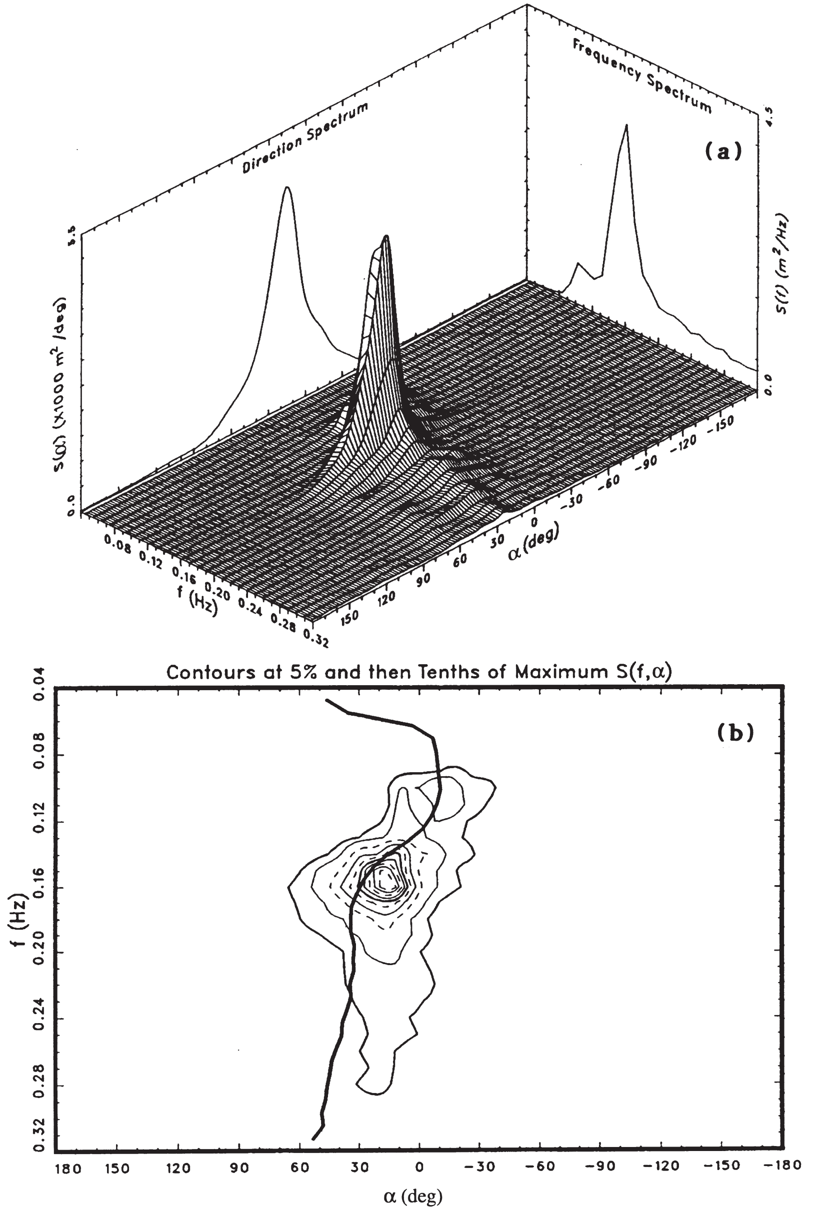

Fig. 6. Frequency–direction spectrum for the record on 2 February 1994 at 1300 hours for the full array using IMLE method. In part (b) the thick curve down the page gives the wave direction (mean of 15 redundant estimates) computed using Esteva's method.

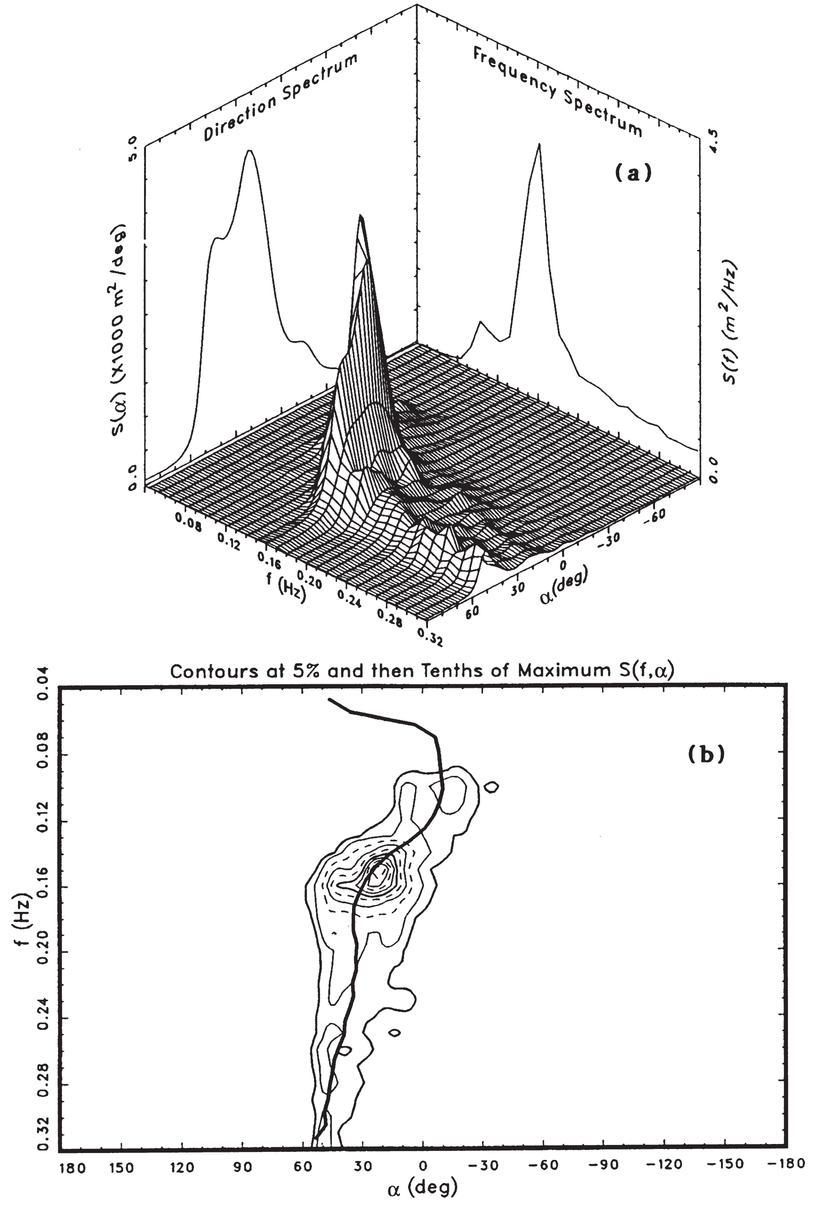

Fig. 7. Frequency–direction spectrum for the record on 2 February 1994 at 1300 hours for the linear (longshore) part of array using IMLE method. In part (b) the thick curve down the page gives the wave direction (mean of 15 redundant estimates) computed using Esteva's method.

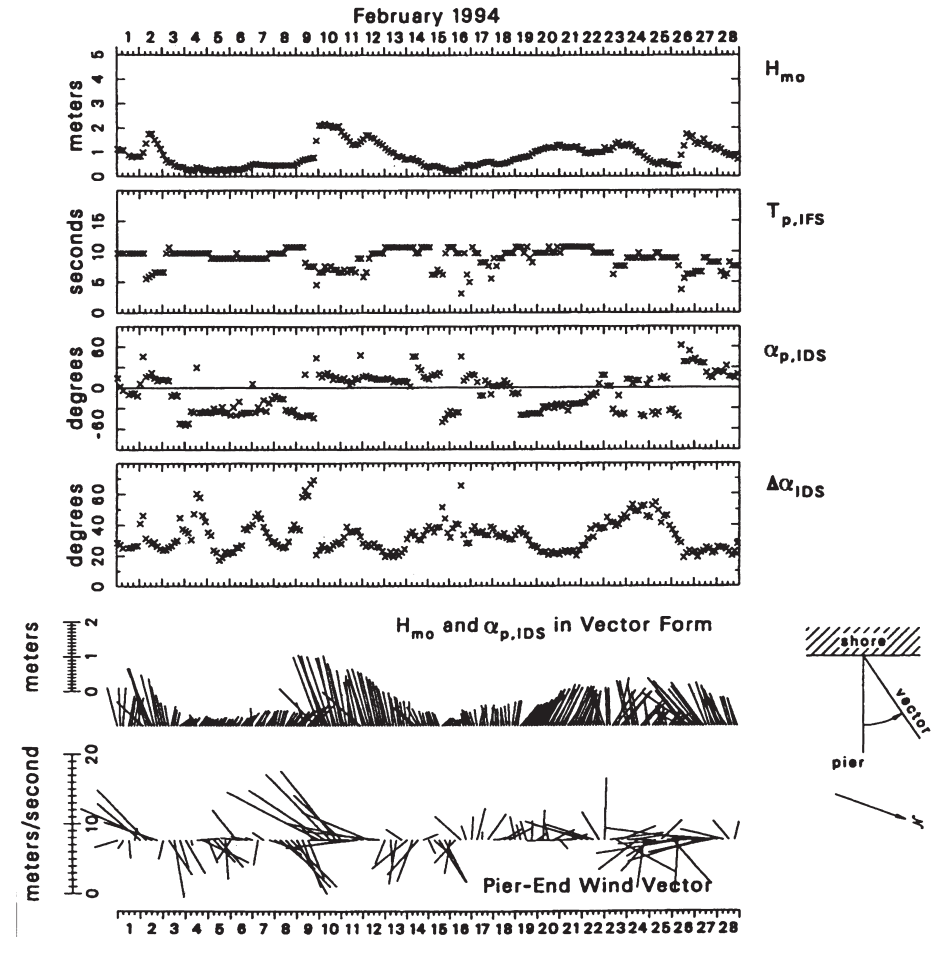

Fig. 8. Bulk parameters derived by the CERC using the IMLE method. The period of interest is 1–6 February. H_{mo} is the significant wave height, $T_{p,IFS}$ is the peak period from the integrated frequency spectrum, $\alpha_{p,IDS}$ is the peak wave direction of the integrated direction spectrum, $\Delta\alpha_{IDS}$ is the directional spread parameter of the integrated direction spectrum (from Long and Roughton, 1995).

be present during this time period. As already mentioned the criterion for occurrence of surf beat of remote origin given by Herbers et al. (1995), is that the total (swell) energy in the range 0.04–0.14 Hz $<$ 100 cm^2. This criterion of Herbers et al. (1995) cannot be used for predicting the occurrence of surf beat of remote origin as it requires information that is not readily available. On the contrary the significant wave height, H_{mo}, is a parameter that is always reported. Therefore, our criterion for occurrence of surf beat of remote origin at Duck, $H_{mo} < 0.41$ m, is superior to that of Herbers et al. (1995).

The histogram in Table 2 shows that 61 out of 72 estimates of direction of surf beat observed during 4–6 February 1994 indicate that the surf beat at Duck propa-

Table 2
Histogram of surf beat of remote orgin (spectral periods 64.00, 42.67 and 32.00 s) occuring during 4–6 February 1994 (eight observations per day) giving a total of 72 estimates of surf beat direction. Surf beat directions are up the coast, towards shore measured with respect to the shore normal. The uniform negative sign of surf beat direction has been dropped for convenience

Range (degrees)	Frequency
30–35	7
35–40	9
40–45	14
45–50	19
50–55	4
55–60	8
60–65	3
65–70	3
–	–
–	–
–	–
95–100	1
Total	72

gates towards shore, up the coast at angles of 30–60° with respect to the shore normal. Table 2 in conjunction with Fig. 9, which depicts great circle paths of the surf beat incident at Duck, show that the surf beat of remote origin is largely trans-oceanic in origin. In one case during low wave conditions, the surf beat of remote origin appears to have been generated as close as the West Indies.

Using the modified method of Borgman, we have made 10 redundant estimates of wave direction as a function of frequency for each of the 47 records of measured data at North Carolina. The following 10 gauge pairs 43, 21, 32, 42, 65, 78, 89, 31, 01 and 54; satisfying $D < \lambda/2$ for wave periods above 9 s were chosen. Two gauge pairs, 43 and 21 satisfied the same criterion for periods above 3.5 s. It was found from the analysis of all the 47 records, that the difference between the mean of the 10 redundant estimates using Borgman's method and the 15 redundant estimates using Esteva's method, had a mean of 0.33° and a standard deviation of 2.89° for spectral periods between 14.22 and 4.27 s ($47 \times 22 = 1034$ points)—the close agreement between the results of Esteva's method and Borgman's method occurs by chance only because the waves of the above period range, in all the 47 records, always propagate towards the shore even when the wind direction is away from shore, so that there is no ambiguity in the Borgman determination.

5. Conclusions

Phase/time/path difference (PTPD) methods of Esteva (1976, 1977) and Borgman (1974) as modified by us can be successfully used for computing wave direction as a function of frequency from actual field measurements using polygonal and linear

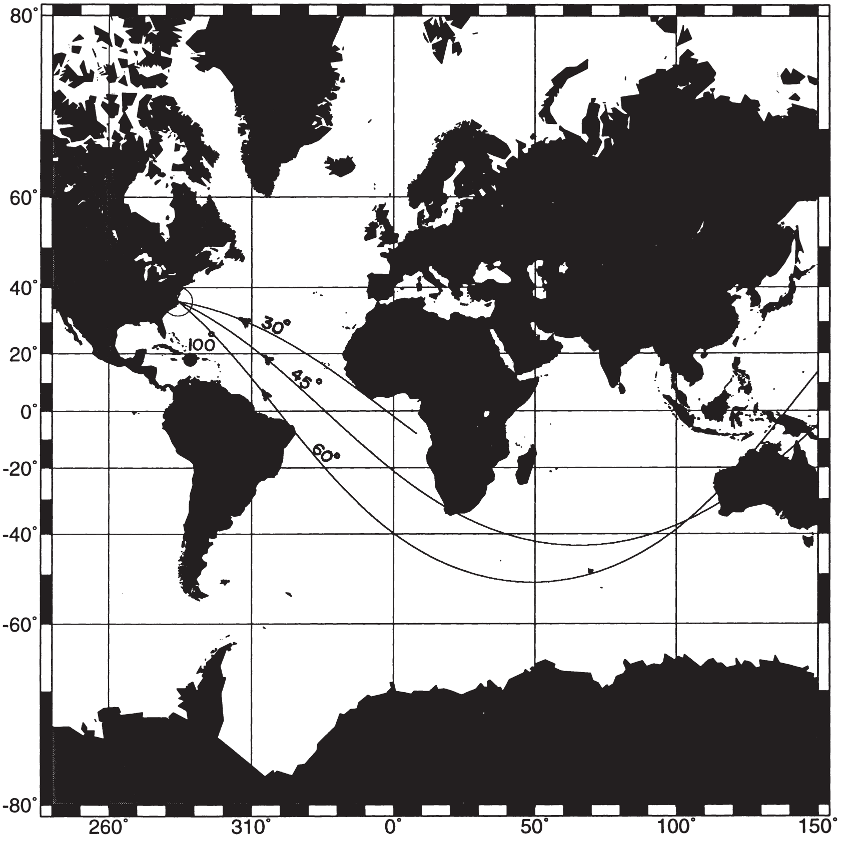

Fig. 9. Great circle paths of surf beat of remote orgin incident at Duck, North Carolina, USA. The direction of surf beat is towards shore, up the coast at the given angle with respect to the shore normal.

arrays respectively in case of swell, sea and surf beat as the directional spread is generally unimodal. PTPD methods fail in case of multimodal directional spreads, which can be easily spotted from the large value of the standard deviation of redundant estimates of wave direction. We have shown that PTPD methods are adept in describing the propagation of waves generated by distant storms (swell); locally generated waves (sea); infra gravity waves (surf beat) locally generated by energetic incident well; and surf beat of remote origin occurring when very low swell energies are present.

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