Evaluation and Performance Enhancement of a Pressure Transducer under Flows, Waves, and a Combination of Flows and Waves*

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ABSTRACT

The performance of a pressure transducer, with its inlet attached to differing hydromechanical front ends, has been evaluated in flow flume and wave flume experiments in which laminar and turbulent flows, and regular progressive gravity waves and combinations of flows and waves, were generated. For steady laminar flows, and for waves propagating on quiescent waters, the transducer's performance improved when the inlet was at the center and flush with a large, thin, and smooth circular horizontal end plate. This enhancement is likely to have been achieved because of the isolation of the pressure inlet from the separated flows and vortices generated by the transducer housing. Flow disturbances, generated by nearby solid structures, deteriorated the performance of the pressure transducer. However, its performance could be significantly improved by protecting the pressure inlet by a sturdy, curved perforated shield. The dynamic pressure error in this case was 2 mb at 100 cm s^{-1}, compared to 8 mb in the absence of the shield. For turbulent flows less than 100 cm s^{-1}, a pair of thin, circular, parallel plates, with a diameter three to four times that of the transducer housing and separation equal to the housing diameter, led to a much improved horizontal azimuthal response. At this speed the spread in the dynamic pressure, ΔP, was less than 1 mb compared to 6 mb without a plate. Beyond this speed the transducer's horizontal azimuthal response deteriorated faster. For combinations of waves and flows a relatively small ΔP was found. This result is of special significance to tidal measurements of coastal waters, in which waves propagate on tidal currents.

1. Introduction

Sea level measurements are subject to various errors, primarily due to flow, waves, and change in the density of water in the vicinity of the gauge. Some of these issues, related to stilling-well gauges, have been initially addressed by Noye (1974) on theoretical grounds, and subsequently addressed by Shih and Baer (1991) on the design aspect. This design has been employed by the U.S. National Ocean Service in its tide and water-level measurement network. Density effects have been addressed by Lennon (1992) and Joseph et al. (1997). Although the stilling well is essentially a pressure system, its dynamics may not be similar to that of a pressure transducer. For this reason, experiments have been conducted separately on a pressure transducer, which is popularly used in sea level measurements. The influence of suspended sediments on the performance of a pressure transducer has been addressed by Joseph et al. (1999). In this paper the experimental performance of a pressure transducer under flows, and a combination of flows and waves, is examined.

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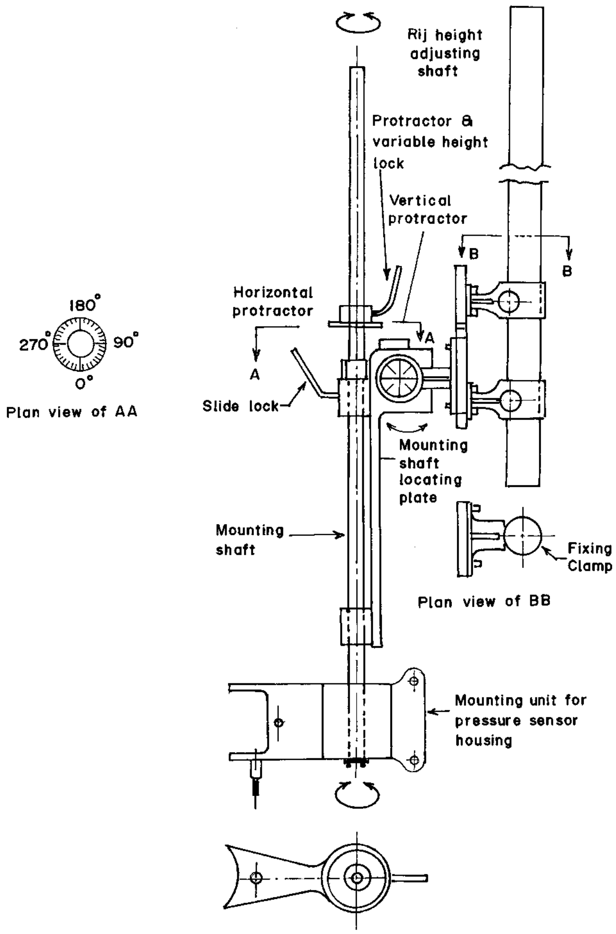


FIG. 1. Support mechanism for pressure transducer housing.

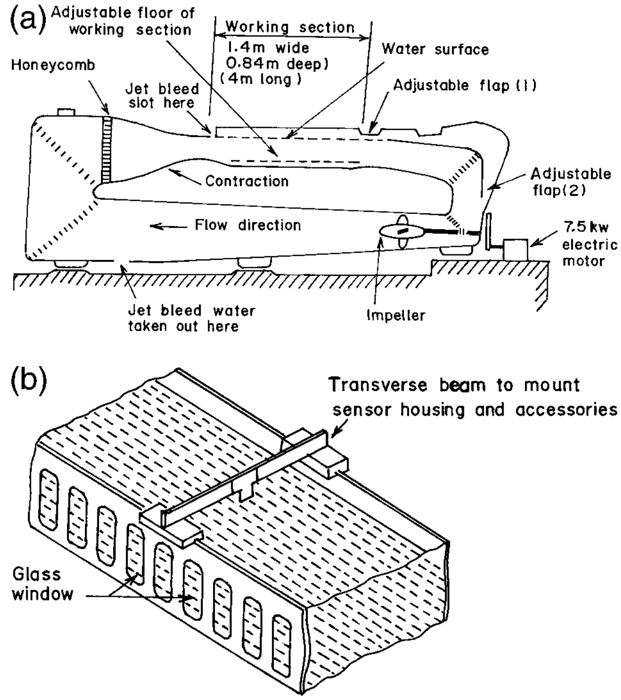


FIG. 2. (a) Schematic diagram of flow flume. (b) Working section of flow channel.

2. Description of experiments

A Digiquartz differential shallow water pressure transducer (0–30 psi), mounted inside a standard Aanderaa pressure housing (of diameter D equal to 12.8 cm), was used for the experiment. To reduce the effects of turbulent wakes shed from the pressure housing, the transducer's positive port was extended to the center of a flat horizontal circular plate. The edge of the plate was rounded to reduce flow separation and shedding. The protruding stainless steel tube (2.5 cm long) was filled with silicone oil to prevent trapping of air inside the pressure inlet. The transducer's accuracy under quiescent conditions is 0.01%. The negative port was attached to one end of a nylon tube; the other end could be mounted at a suitable location in the air. The pressure transducer with its housing was rigidly mounted on a support mechanism (Fig. 1), which could be rotated through 360° about the axis of the support rod. A protractor, mounted on a plane, which is perpendicular to the axis of the support rod, was used for measurement of horizontal azimuthal directions of the transducer. The pressure inlet was located at the bottom portion of the transducer housing to minimize disturbances from the support mechanism.

Flow flume experiments were conducted at the University of Liverpool, United Kingdom. This test facility (Fig. 2a) contained about 90 000 L of water, which was driven around the circuit by an impeller powered by a motor. At the top section of the flume there was an open working section through which the water flowed at speeds controlled from 0.03 to 6.4 m s^{-1}. The flow velocity was uniform across both the width and length of the working section over the full range. A horizontal beam across the working section of the channel (Fig. 2b) permitted mounting the pressure transducer's support mechanism at the middle of the working section. Glass windows on the vertical sides permitted visual monitoring of the flow behavior.

Wave flume experiments were conducted at the wave/tow-tank facility (Fig. 3) at the Institute of Oceanographic Sciences, United Kingdom. The tank was 53 m long by 1.8 m wide and filled with water to a depth of 1.8 m. A plunge-type, wedge-shaped wavemaker fitted at one end of the water tank was capable of producing 0.4–2-s wave periods, with maximum height of 50 cm. An energy absorbing ''beach'' situated at the opposite end of the tank minimized wave reflections.

The datalogger used in the present experiment was the one routinely used in the deep-sea bottom pressure recorders at the Proudman Oceanographic Laboratory (Spencer and Foden 1996). In the present experiment the transducer and the datalogger were continuously

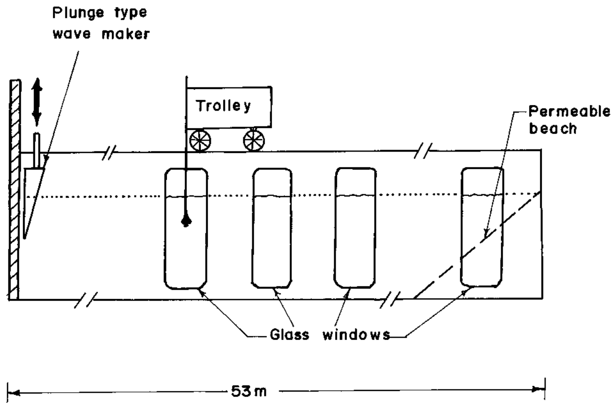


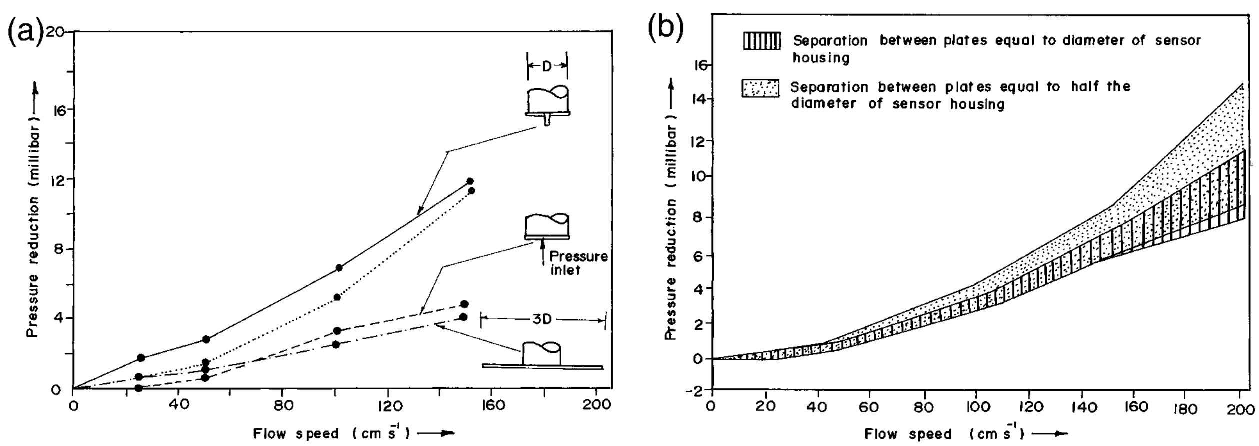
FIG. 3. Schematic diagram of wave/tow-tank facility.

powered, and time-averaged (28.125 s) pressure signal was displayed. During each set of experiments the response of the pressure transducer was first monitored in still water.

3. Experimental results

With a protruding pressure inlet, the pressure transducer exhibited a large ΔP with increasing flow speeds (Fig. 4a). As expected from Bernoulli's theory, ΔP exhibited a square law relationship with respect to the flow speed. A significant improvement in the response of the pressure transducer to a flow field, compared to that of a protruding pressure inlet, has been achieved when the pressure inlet remained flush with the horizontal plane of the end plate. The pressure reduction in this case, at a flow speed of 50 cm s^{-1}, was less than 1 mb. This corresponded to an underestimation in water level of less than 1 cm. Addition of a parallel plate, supported by three equally spaced cylindrical stand-offs (8-mm diameter) was also considered. As stand-offs have been found to alter the flow field in their vicinity (Joseph and Desa 1994), the azimuthal response of the transducer with changing flow direction has also been investigated. Locating the pressure inlet at the center of a large circular plate, whose diameter was three times that of the pressure transducer housing, yielded a significant improvement in the response of the pressure transducer, especially at large flow rates (Fig. 4a). However, the trend in the spread of the azimuthal response for plate separations of D and $D/2$ were similar to those for plate diameter of D (Fig. 4c).

The advantage of locating the pressure inlet at the center of a large, thin circular plate is that the pressure inlet will be free from the influences of the modified velocity field around the pressure transducer housing.



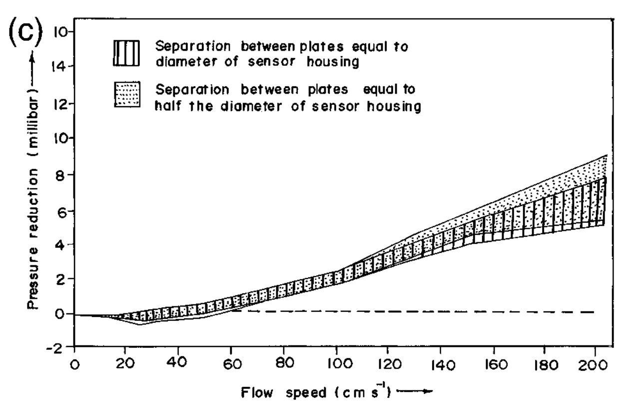


FIG. 4. (a) Response of a pressure transducer to a steady laminar flow field for different hydromechanical front ends (see inset). Dotted line shows the dynamic pressure corresponding to the mainstream flow. (b) Spread in horizontal azimuthal response of a pressure transducer to a steady laminar flow field for small parallel plates. (c) Spread in horizontal azimuthal response of a pressure transducer to a steady laminar flow field for large parallel plates.

Because the plate is thin, the pressure inlet located at its center will be free also from the effects of separated flows and vortices, unless the flow is at some angle of attack. A horizontally placed flat, thin plate, with its edge rounded or chamfered, and whose diameter is sufficiently larger than that of the transducer housing, acts as a barrier that reduces the disturbances generated by the body of the pressure transducer.

a. Response of a pressure transducer to turbulent flows

The output of a pressure transducer mounted near a piling can be influenced by the turbulence generated by the latter. The experiments reported here were performed with the pressure housing mounted in the neighborhood of a cylindrical vertical piling model whose diameter was twice that of the pressure transducer housing. The top end of the piling extended 5 cm above the water surface and its bottom end extended below that of the pressure housing by a distance equal to half the diameter of the pressure transducer housing. Different experimental settings are shown in Fig. 5. Pressure reduction as a function of flow speed for different configurations is shown in Figs. 6a–c.

The horizontal azimuthal angular response of the pressure transducer for different flow arrival angles (Fig. 6a) indicated that the effect of having a piling in the vicinity of the pressure transducer housing is an increase in ΔP as compared to that in the absence of a piling. The value of ΔP was minimum for $\theta = 180°$ and maximum for $\theta = 90°$ and 135°, indicating that the response of a pressure transducer is sensitive to the directions of flow arrival, especially at large flow speeds.

Improvement in the performance (i.e., lesser value of ΔP) of a pressure transducer mounted on a cylindrical piling was sought to be achieved by placing a stainless steel perforated curved shield (holes of 7/16-in. diameter and 5/8-in. pitch) at different distances from the transducer housing. A perforated shield placed 2 cm away from the transducer's housing degraded the performance of the pressure transducer, probably as a result of the swirling motion generated by the orifices of this shield. The transducer's performance improved (Fig. 6b) when the shield was placed at a distance of 2D from the periphery of the transducer housing (Fig. 5c). Mounting the perforated shield at a distance of 4D did not yield any additional improvement.

When the pressure inlet was located at the center of a horizontal, thin circular plate of diameter 3D, and flush with the plate surface, the improvement was similar to that obtained from placing a perforated shield at a large distance in front of the pressure transducer housing. The transducer with a single-plate front end, such as this, was found to be sensitive to different flow arrival angles; the azimuthal response had a spread of approximately 2 mb. Use of a second identical plate, separated from the first by a distance equal to D, yielded a smaller spread in the horizontal azimuthal response in the flow range of 0–100 cm s^{-1}; but a larger spread when the flow speed exceeded 100 cm s^{-1} (Fig. 6c).

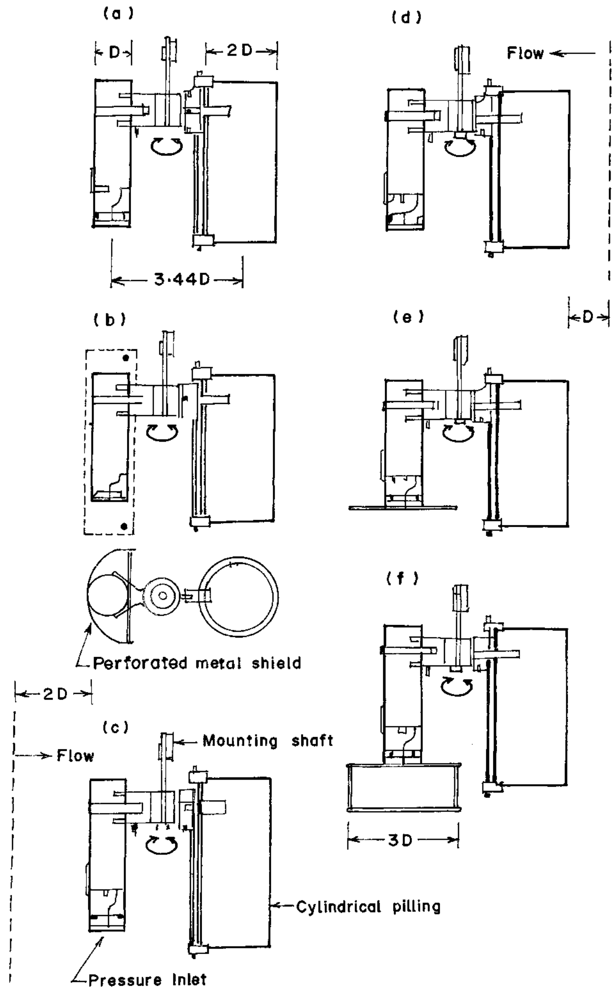
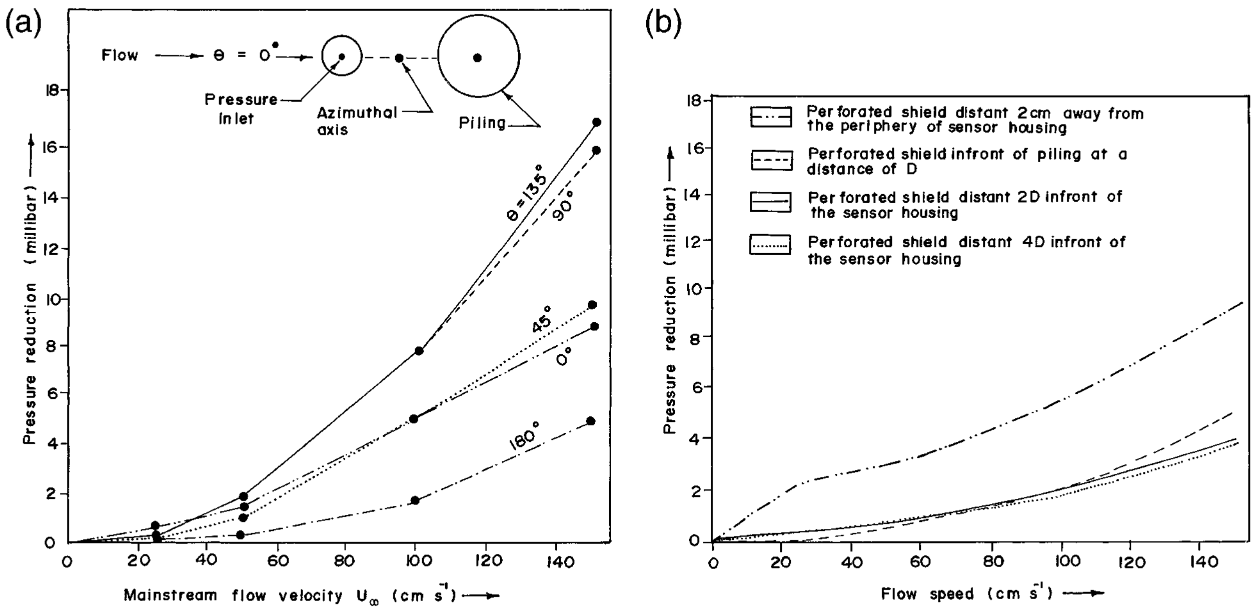


FIG. 5. Different experimental settings for the pressure transducer housing mounted near a cylindrical piling.

b. Response of a pressure transducer to progressive gravity waves propagating on quiescent water

During each set of measurements the response of the pressure transducer was first monitored in still water and then at different wave heights ranging from 15.24 to 30.48 cm. The wavemaker was started from rest at the beginning of each test and at least 5 min were allowed to elapse before the measurements were initiated. As the value of ΔP is a function of the square of the wave height (Stokes' second-order theory), it is more important to investigate the performance of the pressure transducer in relation to the wave height, although wave period is also important to a lesser extent. Consequently, all the performance evaluation experiments were conducted for different wave heights and a constant wave



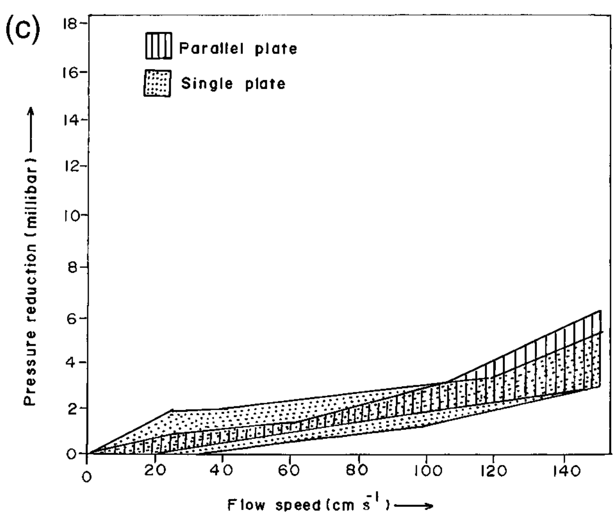


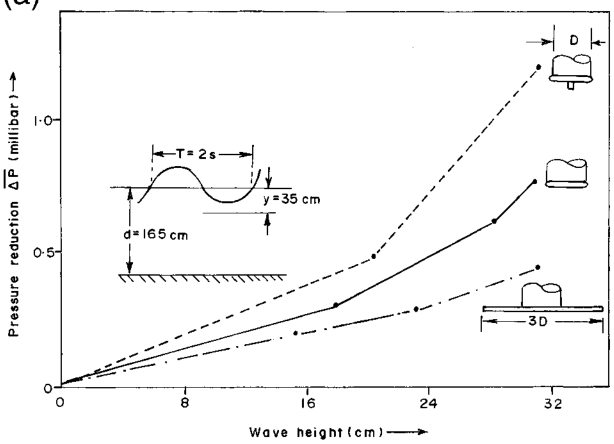
FIG. 6. (a) Performance of a pressure transducer to various mainstream flows for different relative orientations of its housing and a cylindrical vertical piling. (b) Effect of a perforated curved shield in the vicinity of the pressure transducer housing on the performance of the pressure transducer. (c) Effect of a single large end plate and a parallel pair on the performance of the pressure transducer.

period of 2 s. Figure 7 shows the dependence of ΔP on wave height for several configurations of front ends. Results of the experiments using the protruding pressure inlet have indicated that, as the wave height increased the time-averaged pressure output of the transducer was less than that of the still water pressure at a given depth of deployment. Results of the experiments, with the pressure inlet flush with and located at the center of the horizontal plane of the end plate, revealed the same trend. However, the pressure reduction in this case was significantly smaller than that in the case of a protruding pressure inlet. Addition of a similar circular parallel plate, supported by three equally spaced cylindrical stand-offs (8-mm diameter) and separated by a distance D, further reduced the value of ΔP for most of the azimuthal directions, even being negative in some directions (Fig. 7b). This tendency was strengthened when the separation between the plates was reduced to $D/2$ (Fig. 7c). Locating the pressure inlet at the center of a larger horizontal circular plate, whose diameter was $3D$, gave rise to a significant improvement in the performance of the transducer (i.e., lower value for ΔP) compared to all other hydromechanical front-end designs. Addition of a second circular plate of diameter $3D$ separated from the first by D and ($D/2$) yielded deteriorating effects on the performance of the pressure transducer given the above wave conditions (Fig. 7c).

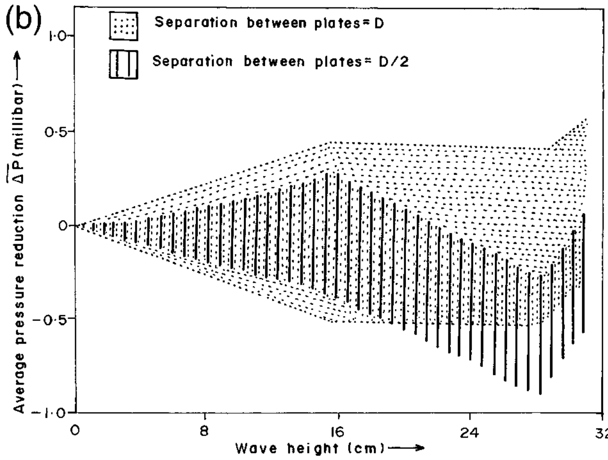
c. Response of a pressure transducer to progressive waves riding on positive and negative currents

In coastal waters the waves, in general, propagate on currents rather than on quiescent waters. As waves propagate onto an opposing current they tend to shorten in

(a)



(b)



(c)

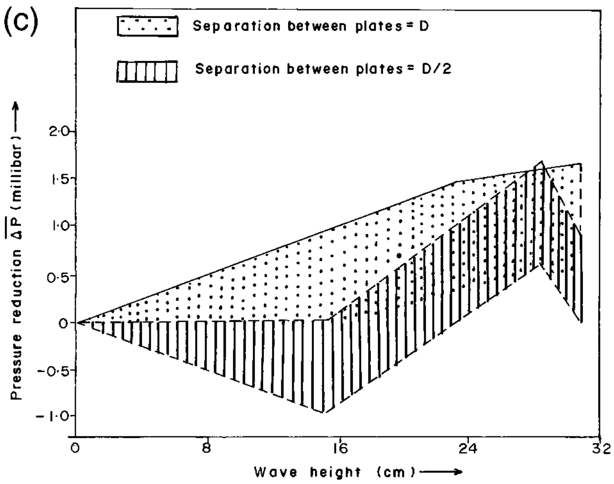


FIG. 7. (a) Response of a pressure transducer to progressive gravity waves propagating on quiescent water for the hydromechanical front ends shown in the inset. (b) Spread in horizontal azimuthal response of a pressure transducer to progressive gravity waves propagating on quiescent water for small parallel plates of diameter *D*. (c) Spread in horizontal azimuthal response of a pressure transducer to progressive gravity waves propagating on quiescent water for large parallel plates of diameter 3*D*.

wavelength and increase in height. Changes in wavelength and wave height affect the value of ΔP. Current-induced modifications of waves also affect the pressure field accompanying the waves, causing an error in the measurement of *P* (Herchenroder 1981). In this section effects of waves riding on positive and negative currents, based on experimental results, are considered. Trolley speed was taken as negative when the trolley moved in the wave direction and considered positive when its motion was against the wave direction.

In the case of a protruding pressure inlet, the combined effect of waves riding on positive and negative flows was to give rise to larger ΔP values as compared to those when waves were absent (Fig. 8). In the absence of a protrusion, the experimental results (Fig. 9) were, in general, similar in trend but smaller to those for a protruding pressure inlet.

Locating the pressure inlet at the center of a horizontal, thin circular end plate of diameter 3*D* yielded a very significant improvement in the performance of the transducer (i.e., lower ΔP values). In some cases, the combined effect of waves and flows even yielded negative values for ΔP (Fig. 10). Use of a second identical plate, separated from the first by a distance equal to *D*, exhibited a similar trend, showing an improved performance at large wave heights (Fig. 11).

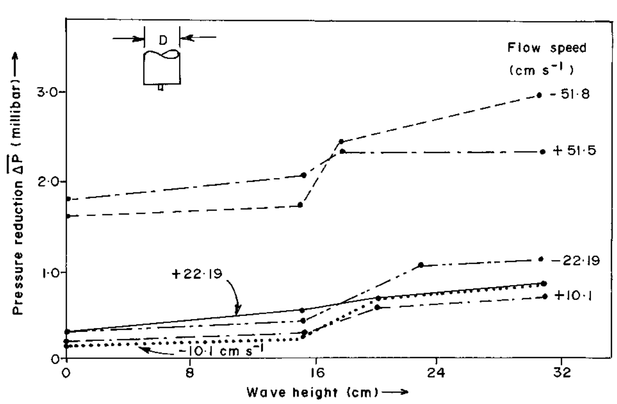


FIG. 8. Response of a pressure transducer for waves riding on positive and negative currents for a protruding inlet (see inset).

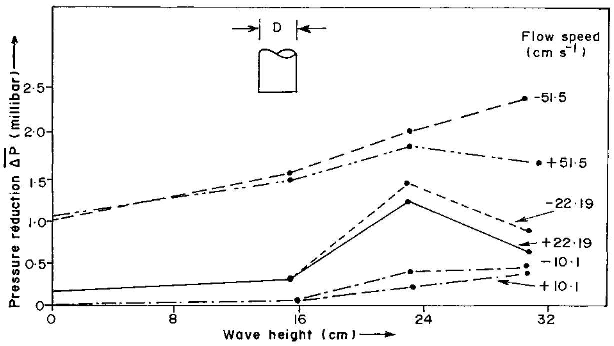


FIG. 9. Response of a pressure transducer to waves propagating on positive and negative currents for pressure inlet located at the center of the end plate of the transducer housing and flush with the plate surface (see inset).

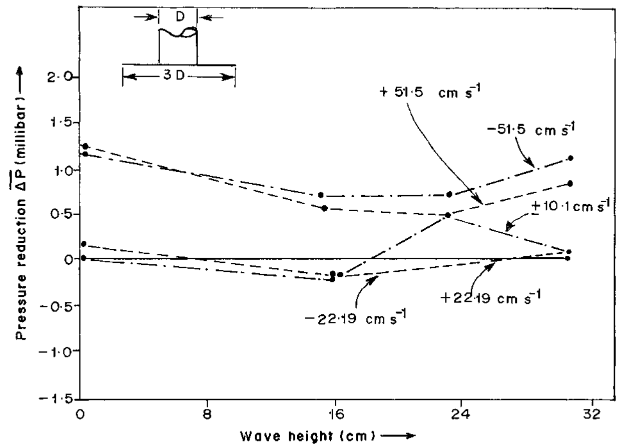


FIG. 10. Response of a pressure transducer to waves propagating on positive and negative currents for pressure inlet located at the center of a large end plate (see inset).

4. Discussion of results

Placement of a pressure transducer or probe in a flow field causes streamlines to deform and hence changes the static pressure at the transducer from the free-stream value. The value of the dynamic pressure ΔP is proportional to the dynamic head, as evidenced in the Bernoulli's equation. The flow can be laminar or turbulent, depending on the details of the pressure housing, supporting structure, etc. The output of a pressure transducer is also influenced by wave-induced oscillatory flows and radiation stress in the vicinity of the pressure inlet. Wave–current interaction also appears to play a role in the performance of a pressure transducer deployed for sea level measurements.

a. Effect of laminar and turbulent flows

It was noted that a protruding pressure inlet exhibited the largest error in the pressure measurement. This protrusion (2.5 cm) was considerably outside the boundary layer ($\approx$3 mm), and it was therefore exposed to the mainstream flow and the separated flow generated from the transducer housing. Figure 4a shows that the observed values of ΔP were consistently higher than the dynamic pressure in the uniform flow range of 0–150 cm s^{-1}. The higher values of the measured ΔP might have been caused by the fact that the protruding pressure inlet was exposed to separated flows in which there was an acceleration of the ambient flow. The observed reduction in ΔP when the pressure inlet remained flush with the end plate of the transducer's housing appears to be due to the pressure inlet being distant from the separated flow and the vortices shed by the transducer housing, as compared to the protruded pressure inlet. In this case the pressure errors for flow speeds of less than 60 cm s^{-1} are approximately equal to the dynamic pressure.

A slight increase in ΔP with the use of a second parallel plate of diameter D is likely to have resulted from the disturbances generated by the stand-offs of the parallel-plate assembly. In the case of parallel plates of diameter 3D, small negative values of ΔP were observed for most of the azimuthal angles, when the mainstream flow was 25 cm s^{-1}. Figure 6a shows that the presence of a piling near a pressure transducer has an adverse effect on its performance for most of the flow arrival angles (θ). The ΔP values were comparatively less for flow arrival angles of 0° and 180°, and maximum at 90° and 135°. An analysis revealed that the experimental values of ΔP (i.e., ΔP_{expt}) were similar to the calculated values of ΔP (i.e., ΔP_{cal}) for $\theta < 90°$ (Fig. 12). However, ΔP_{expt} values were at variance with ΔP_{cal} for θ between 90° and 180°. In this analysis the values of ΔP were calculated based on the modified flow in the vicinity of the pressure inlet arising from the influence of the cylindrical piling. The difference between ΔP_{expt} and ΔP_{cal} as a function of flow arrival angle θ for various flow speeds is shown in Fig. 13. The values of ΔP_{expt} were consistently higher than those of ΔP_{cal} in the angular range 70°–150°.

In the present experiment, for flow speeds up to 117

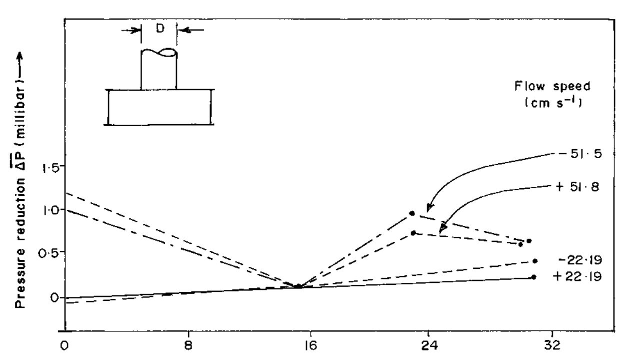


FIG. 11. Response of a pressure transducer to waves propagating on positive and negative currents for pressure inlet attached to large parallel plates (see inset).

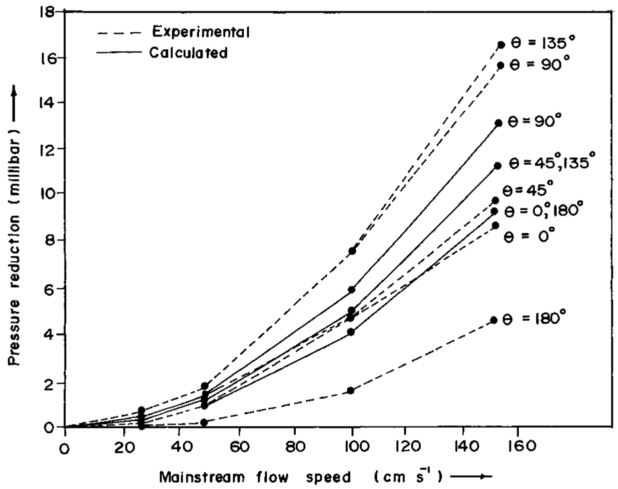


FIG. 12. Experimental and theoretical values of ΔP for various flow speeds and directions, and for a pressure transducer mounted near a cylindrical piling as in Fig. 5a.

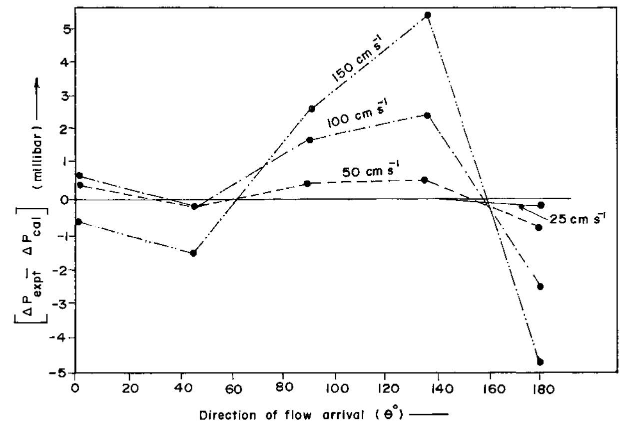


FIG. 13. Dependence of difference between experimental and theoretical values of ΔP on flow directions.

cm s^{-1} the Reynolds number was less than the critical value of 3×10^5, beyond which the flow became turbulent in the vicinity of a cylinder. In this case, the steady flow assumption is not applicable. A lower value for ΔP_{expt} at 180° compared to ΔP_{cal} can be explained on the basis of a lower pressure field in the vicinity of the leeward stagnation point of the piling. Thus, the complicated flow field around the piling and the location of the pressure housing with respect to the flow field appears to be the controlling factors that dictate the value of ΔP.

A perforated shield, located sufficiently away from the pressure inlet, improved the transducer's performance in a flow field significantly. A parallel-plate geometry also worked effectively in reducing the dynamic pressure. However, the reasons for the effectiveness of these devices, in reducing the dynamic pressure, remain poorly understood.

b. *Effect of waves, and a combination of flows and waves*

Figures 7b,c indicate that a parallel-plate front end has a profound influence on the horizontal azimuthal response of a pressure transducer to a progressive gravity wave field, yielding negative values of ΔP at some azimuthal directions. An important role of horizontal parallel plates of the present experimental setup appears to be to modify the wave-induced elliptical motions of water particles in the vicinity of the pressure inlet. For all the wave heights, the vertical heights $2B$ of the minor axes of water particle ellipses in the wave flume (Shore Protection Manual, U.S. Army CERC 1977) were greater than the vertical separation between the horizontal plates, in the case of plate separation equal to $0.5D$ (D = 12.8 cm). For most of the wave heights the value of the plate separation was smaller than the distance $2B$. This might have resulted in compression of the elliptical orbits of water particle motions within the confines of the horizontal parallel plates. Such deformations, together with mutual interaction between the vortices around the three stand-offs, must have given rise to radiation stresses (Longuet-Higgins and Stewart 1964) favorable to an increase in the pressure field (i.e., negative ΔP) and thus resulted in the observed enhancement.

A prominent feature, noticed in the case of parallel plates of diameter D and $3D$ alike, was an increased spread in the horizontal azimuthal response of the transducer for a plate separation of $0.5D$, as compared to a separation of D. Another notable feature, in the case of large parallel plates (diameter = $3D$), was that ΔP was always positive when the separation between the plates was equal to D. However, the spread in the azimuthal response was ≈1.5 times that in the case of small diameter plates (diameter = D). In general, use of large diameter parallel plates had a deteriorating effect on the performance of the transducer, as compared to the use of small diameter parallel plates. This result in the case of waves propagating on quiescent water is in marked contrast with the response of the transducer to a flow field. However, it must be observed that generation of negative ΔP has a significant advantage when waves ride on a current.

c. *Response of a pressure transducer to waves riding on currents*

With the use of a parallel-plate geometry at the pressure inlet, the flow in the vicinity of the pressure inlet assumes the characteristics of oscillatory plane Poiseuille (OPP) flow. This consists of the superposition of the steady parabolic axial flow velocity profile (arising from steady flow) and an axial oscillatory velocity profile (arising from progressive wave). Studies by Von Kerczek (1982) showed that the sinusoidal OPP flow is more stable than the steady plane Poiseuille flow for a wide range of frequencies and amplitudes of the imposed oscillations. Radiation stress caused by interactions, or any kind of resonant coupling, between the orbital motions in the region bounded by the two parallel plates and the vortices shed from the standoffs, may also have contributed to the observed reduced ΔP values.

5. Conclusions

Flow channel and wave flume experiments have been conducted on a pressure transducer with differing configurations of hydromechanical front ends to evaluate its performance under regular progressive gravity waves, and combinations of flows and waves. Based on flow flume and wave flume investigations the following observations can be made:

1) A protruding pressure inlet is most susceptible to flow- and wave-induced errors.
2) For laminar flows, the inlet of the pressure transducer should be located at the center, and flush with a horizontal end plate (edge rounded or chamfered), of diameter equal to three to four times that of the pressure housing.
3) For turbulent flows, and a combination of flows and waves, a pair of thin circular parallel plates (edges rounded or chamfered) of diameter three to four times that of the housing, and separation equal to the housing diameter, is preferred.
4) A perforated shield is an equally effective device to improve the performance of a pressure transducer deployed in a flow field.

For a parallel-plate front end, a reduced value of ΔP has been observed for combinations of waves and currents, as compared to currents alone. This result appears because the observed ΔP is a sum of current-induced positive ΔP and wave-induced negative ΔP. Thus, a horizontal parallel-plate front-end mechanism is more effective in the presence of combinations of waves and flows than solely for flows or for waves. Further studies of tilt response of the transducer are necessary, as these have important implications to open ocean measurements in terms of precision and accuracy in such measurements.

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