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Value at Risk: Issues and implementation in Forex Market in India

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Abstract: Value-at-Risk (VaR) has been widely promoted by the Bank for International Settlement (BIS) as well as central banks of all countries as a way of monitoring and managing market risk and as a basis for setting regulatory minimum capital standards. The revised Basle Accord, implemented in January 1998, makes it mandatory for banks to use VaR as a basis for determining the amount of regulatory capital adequate for covering market risk. Foreign exchange forms a major part of banks' holding and hence are subject to risk. We have adopted three categories of VaR methods, viz., Variance-Covariance (Normal) methods including Risk-Metric, Historical Simulation (HS) and Tail-Index Based approach. We have used the daily exchange rate data from March 1, 1993 to October 8, 2003 for our analysis. Empirical results show that most of the models are failing in 1 500 rolling window while the full sample data is over estimating the VaR.

Key Words: Market Risk, Value-at-Risk, VaR Models.

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1. Introduction

Value at Risk (VaR) models plays a core role in the risk management of today's financial institutions. A VaR model measures market risk by determining how much the value of a portfolio could decline over a given period of time with a given probability due to the change in the market price of an asset. A number of VaR models are used with all having the same aim to measure the size of potential future losses at a given confidence level. The way the losses are estimated can vary. Models differ in fact in the way they calculate the density function of future profits and losses of current positions, as well as the assumptions they rely on. The VaR analysis originated with the variance-covariance model introduced by JP Morgan, RiskMetrics (1993). The variance-covariance approach to calculate risk can be traced back to the early days of modern portfolio theory of Markowitz (1959), upon which most today risk managers have been educated. This is why this type of VaR models have had a lot of appeal in the early days.

The two most important components of VaR models are the length of time over which market risk is to be measured and the confidence level at which the market risk is measured. These choices affect the nature of VaR models. Financial institutions are subject to different types of risk, such as, business risk, strategic risk, financial risk and financial risk is one that is caused by movements in financial markets (van den Goorbergh, 1999). The literature distinguishes four major categories of financial risk, viz., credit risk, operational risk, liquidity risk and market risk. Credit risk generally relates to the potential loss due to the default on the part of the counterparty to meet its obligations at designated time. It has three basic components: credit exposure, probability of default and loss in the event of default. Operational risk takes into account the errors that can be made in instructing payments or settling transactions, and includes the risk of fraud and regulatory risks. Liquidity risk is caused by an unexpected large and stressful negative cash flow over a short period. If a firm has highly illiquid assets and suddenly needs some liquidity, it may be compelled to sell some of its assets at a discount. Finally, market risk estimates the loss of an investment portfolio due to the changes in prices of financial assets and liabilities (market conditions).

Monitoring market risk assumes importance to banks and financial institutions, as the values of investment portfolios they hold undergo changes as and when market conditions change. Measuring market risk is important from the viewpoint of devising risk management strategy and for assessing total financial risk (which includes all different types of risks) of an investment portfolio held by a bank or financial institution. There is a need to provide capital charge for this category of risk also so that the banks/institutions remain in business in adverse market conditions. Recognizing this point the Bank for International Settlements (BIS) has included market risk as a part of the total risk for which capital has to be provided by a bank[1].

In recent years, Value at Risk (VaR) has become the standard measure that financial analysts use to quantify the market risk. VaR is commonly defined as the maximum potential fall in value of a portfolio (i.e. loss in portfolio) of financial instruments with a given probability over a certain horizon. In simpler words, it is a number that indicates how much a financial institution can lose with probability, say p, over a given time horizon. The great popularity that this instrument has achieved among financial practitioners is essentially due to its conceptual simplicity: VaR reduces the (market) risk associated with any portfolio to just one number that is the loss associated with a given probability and horizon.

VaR measures can have many applications. It evaluates the performance of risk takers and satisfies the regulatory requirements. VaR has become an indispensable tool for monitoring risk and an integral part of methodologies that allocate capital to various lines of business. Today regulators all over the globe have been forcing institutions to adopt internal models and calculate the required capital charge based on VaR methodologies. In particular, the Basel Committee on Banking Supervision (1996) of the BIS imposes requirements on banks to meet capital requirements based on the VaR estimated through internal model approach. Under this approach, regulators do not provide any specific VaR measurement technique to their supervised banks – the banks

[1] Amendment to the Capital Accord to incorporate Market Risk, BIS document, Basel Committee on Banking Supervision, January 1996.

are free to use their own model. But to eliminate the possible inertia of supervised banks to underestimate VaR so as to reduce the capital requirements, BIS has prescribed certain minimum standard of VaR estimates and also certain tests, such as backtesting, of VaR models. If VaR model of a bank fails in backtesting, a penalty is imposed resulting to higher capital charge.

Thus, providing accurate estimates of VaR is of crucial importance for all stakeholders. If the underlying risk is not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability or the financial stability of the institutions. A b ank would like to pick up a model that would generate as low VaR as possible but pass through the backtesting.

Statistical models however impose strong assumptions about the underlying data. For example, the density function of daily returns follows a theoretical distribution (usually normal) and have constant means and variances. The empirical evidence about the distributional properties of speculative price changes provides evidence against these assumptions, Kendall M (1953), Mandelbrot (1963). Risk managers have also seen their daily portfolios profits and mainly losses to be much larger to those predicted by the normal distribution. The RiskMetrics VaR method has two additional major limitations. It linearises derivative positions and it does not take into account expiring contracts. These shortcomings may result in large biases, particular for longer VaR horizons and/or portfolios overweighed with short out-of-the money options.

Risk managers have begun to look at simulation techniques to overcome the limitations of the variance-covariance approach. Simulation is used to generate pathways or scenarios for linear positions, interest rate factors, FX rates, etc and then value all positions at each scenario. The VaR is therefore calculated from the density function of the simulated portfolio values. The use of Monte-Carlo simulation is widely spread between the financial institutions around the globe. This method also is not lacking of severe criticisms. Firstly, the generation of the scenarios is based on random numbers drawn from a theoretical distribution, often normal, which does not always conforms with the empirical distribution of the data. Furthermore, to maintain the multivariate properties of the risk factors when generating scenarios, historical correlations are used. During market crises, when most correlations tend to increase instantly, a Monte Carlo system is likely to underestimate the possible losses.

Recognising the fact that most asset returns cannot be described by a theoretical distribution, an increasing number of financial institutions are using historical simulation. Here, each historical observation forms a possible scenario. A number of scenarios is generated and in each of them all current positions are priced. The resulting portfolio distribution is more realistic since it is based on the empirical distribution of risk factors. This method has still some serious setbacks. The historical returns that are used as random numbers are not i.i.d. Thus the produced VaR value will be biased. During high volatile market conditions the historical simulation will underestimate risk. Furthermore, historical simulation uses constant implied volatility to price the options under each scenario. Some positions that may be appear well-hedged under the constant implied volatility hypothesis, may become very risky under a more realistic scenario.

From a statistical point of view, VaR estimation entails the estimation of a quantile of the distribution of returns. Though, t here has been voluminous work done on VaR in financial market all over the world, the task of estimating/forecasting VaR still remains challenging. The major difficulty lies in modelling/approximating the return distribution, which generally is not normal (being skewed and/or having fatter tails than normal distribution due to asymmetry, volatility clustering, etc.). Available VaR models can be classified into four broad categories: the historical simulation method, the Monte Carlo simulation method, modelling return distribution (including the variance/covariance method, which assumes normality of the return distribution, and methods under Extreme Value Theory (EVT). All these VaR estimation methods adopt the classical approach: they deal with the statistical distribution of time series of returns.

The objective of the paper is to look at foreign exchange market in India and study various VaR methods using the Rupee-Dollar exchange rate data to understand which method is best suited for Indian system. The paper has been designed as follows: Section

2 presents a brief review of literature on the subject, Section 3 discusses the theoretical and methodological issues concerning VaR, Section 4 focuses on data and construction of portfolio, Section 5 discusses empirical results and Section 6 concludes.

2. Literature Review:

The BIS (1996) document states (pg 45) that each bank must meet, on a daily basis, a capital requirement expressed as the higher of (i) its previous day's VaR number measured according to the parameters specified and (ii) an average of the daily VaR measures on each of the preceding sixty business days, multiplied by a multiplication factor. The multiplication factor will be set by the individual supervisory authorities on the basis of their assessment of the quality of the bank's risk management system, subject to an absolute minimum of 3. Banks are required to add to this factor a plus directly related to the ex-post performance of the model, thereby introducing a built-n positive incentive to maintain the predictive quality of the model. In India RBI has indicated this number to be 3.3 in its circular to primary dealers. The RBI instruction has been exactly in line with the primary BIS document.

Lopez (1996) and Best (1999) and Jorion (2001) interpreted the BIS guidelines on market risk through the equation:

$$MRC_{mt+1} = Max\left[VaR_{mt}(10,1); S_{mt} * \frac{1}{60}\sum_{i=0}^{59} VaR_{t-i} \right] + S_{mt}$$

where S_{mt} and SR $_{mt}$ are a multiplication factor and an additional capital charge for the portfolio's idiosyncratic credit risk, respectively. Note that, under the current framework, S_{mt} > 3.

Khindanova and Rachev (1998) interpreted the BIS guidelines on market risk through the equation:

$$C_t = A_t * Max\left[\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}, VaR_t \right] + S_t$$

where C_t is the market risk capital requirement and A_t is the multiplication factor between 3 and 4, S_t is the capital charge for specific risk.

Both these equations have different connotations. We have used the first equation for our VaR estimation.

There has been large volume of literature on VaR methodologies as well as on its implementation. The concept received tremendous response from banks all over the world. Banks management can apply the VaR concept to set capital requirements because VaR models allow for an estimate of capital loss due to market risk (see Hendricks, 1996; Lopez, 1996; Duffie and Pan, 1997; Jackson, Maude and Perraudin, 1997; Jorion, 1997; Saunders, 1999; Friedmann and Sanddrof-Kohle, 2000; Hartmann-Wendels, et al., 2000; Simons, 2000, among others).

The computation of volatility is the most important aspect of any VaR estimation. The volatility estimation should take care of the most stylized facts of any financial asset class – the important ones being fat tailed property, volatility clustering and asymmetry of return distribution. Once these issues are identified in the distribution, then calculating volatility is easy. Today GARCH family models have been increasingly used by researchers to model volatility. An important documentation in this regard has been the J P Morgan's RiskMetrics that applied declining weights to past returns to compute volatility with a decay factor 0.94 which is a variant of IGARCH. Other measures of volatility, which differs from the estimation of return variance, include Garman and Klass (1980), and Gallant and Tauchen (1998), who incorporate daily high and low quotes, and Andersen and Bollerslev (1998) and Andersen, et al. (1999), who use average intraday squared returns to estimate daily volatility.

Several studies such as Danielsson and de Vries (1997), Christoffersen (1998), and Engle and Manganelli (1999) have found significant improvements possible when deviations from the relatively rigid RiskMetrics framework are explored. Choosing an appropriate VaR measure is an important and difficult task, and risk managers have coined the term Model Risk to cover the hazards from working with potentially mis-specified models. Beder (1995), for example, compares simulation-based and parametric models on fixed income and stock option portfolios and finds apparently economically large differences in the VaRs from different models applied to the same portfolio. Hendricks (1996) finds similar results analyzing foreign exchange portfolios. In Indian context, Darbha (2001) made a comparative study of three models – Normal, HS and Extreme Value Theory while studying the portfolio of Gilts held by PDs. Sarma, Thomas and Shah (2003) have studied the VaR model selection in Indian context using the stock market data and came up with a loss function based on opportunity cost of capital in case a bank is to have more capital charge if the model is over estimating VaR numbers. However, a bank has its capital sunk in various assets and it is not holding idea cash or setting aside a specific sum of money to meet the capital requirement and hence assigning an opportunity cost would not be justified. The capital which has been used by the bank to create various assets have been earning returns for the bank. It may be the case that a bank has invested in more liquid assets like gilts to take care of its loss scenarios in case liquidation is warranted.

3. Theoretical Issues

As stated earlier, VaR is the maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence and typically calculated for a one-day time horizon with 95% or 99% confidence level. Holding period is one of the most key elements in VaR estimation and the same is chosen on the basis of time that an organization would take to liquidate its position if the need arises. In a very liquid market, 1-day may holding period seem to be justified while in an illiquid market; it may take more than 10 days to liquidate the portfolio. Hence the capital charge would be different for different holding period.

BIS requires that VaR be computed daily by Banks, using a 99^{th} percentile, one-tailed confidence interval with a minimum price shock equivalent to ten trading days (holding period) and the model incorporate a historical observation period of at least one year. The capital charge for a bank that uses a proprietary model will be higher of (i) The previous day's VaR and (ii) an average of the daily VaR of the preceding sixty business days, multiplied by a multiplication factor. The multiplication factor may be 3 and this may go up if the regulators feel that 3 is not sufficient to account for potential weaknesses in the modeling process.

In the case of PDs, RBI prescribes all these above criteria except that (i) minimum holding period would be thirty trading days; (ii) the minimum length of the historical observation period used for calculating VaR should be one year or 250 trading days. For PDs who use a weighting scheme or other methods for the historical observation period, the "effective" observation period must be at least one year (that is, the weighted average time lag of the individual observations cannot be less than 6 months); and (iii) the multiplication factor is presently fixed at 3.3.

The weaknesses may be due to (a) market prices often display patterns (heteroskedastic) that differs from the statistical simplifications used in modeling, (b) past not being always a good approximation of the future (October 1987 crash happened that did not have parallel in historical data), (c) most of the models take ex-post volatility and not ex-ante, (d) VaR estimations normally is based on end-of-day positions and not take into account intra-day risk, (e) models can not adequately capture event risk arising from exceptional market circumstances. Since VaR heavily relies on the availability of historical market price data on the portfolio to understand its effectiveness, it would be appropriate to use the long historical data to see if the stress conditions can be replicated.

3.1. Basic Statistics Related to VaR

VaR models are characterized by their forecasted distributions of k-period-ahead portfolio returns. To fix notation, let y denote the log of portfolio value at time t. The k-period-ahead portfolio return is $\varepsilon_{t+k} = y_{t+k} - y_t$, .Conditional on the information available at time t, ε_{t+k} is a random variable with distribution f_{t+k} that is $\varepsilon_{t+k} | \Omega t \sim f_{t+k}$. Thus, VaR model m is characterized by f_{mt+k}, its forecast of f_{t+k} VaR estimates are the most common type of forecast generated from VaR models. A VaR estimate is simply a specified quantile of the forecasted return distribution over a given holding period. The VaR estimate at time t derived from model m for a k-period-ahead return, denoted VaR (k, α), is the critical value that corresponds to the lower α percent tail of $f_{mt+k.}$

The portfolio consists of many securities and in our case we are concerned with only foreign exchange rate (Rupee – US Dollar rate). The basic price equation of the portfolio can be written as follows:

$$Price_{portfolio} = w_1 * \Pr_{bond1} + w_2 * \Pr_{bond2} + ... + w_n * \Pr_{bondn} \quad (1)$$

and the return on the portfolio is at time defined as

$$R_{pf,t+1} = \sum_{i=1}^{n} w_i * R_{i,t+1} \qquad (2)$$

where the sum is taken over n securities in the portfolio, w_i denotes the proportionate value of the holding of security *i* at the end of day *t*.

And the variance of the portfolio should be written as

$$\sigma^2_{PF,\,t+1} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i * w_j * \sigma_{ij,\,t+1} = \sum_{i=1}^{n} \sum_{j \, 2}^{n} w_i * w_j * \sigma_{i,\,t+1} \sigma_{j,\,t+1} * \rho_{ij,\,t+1} \quad (3)$$

where $\sigma_{ij,t+1}$ is the covariance and $\rho_{ij,t+1}$ is the correlation between security *i* and *j* on day *t+1* and for $\rho_{ij,t+1} = 1$ and we write $\sigma_{ij,t+1} = \sigma^2_{i,t+1}$ for all i.

The VaR of the portfolio is simply

$$VaR^{p}_{PF,t+1} = \sigma_{PF,t+1} * F_p^{-1} \qquad (4)$$

where F_p^{-1} is the p'th quantile of the rescaled portfolio returns.

When we use HS method, we write the VaR equation as

$$VaR^{p}_{PF,t+1} = -percentile\{\{R_{PF,t+1}\}_{r=1}^{m}, 100p\}$$

3.2. Select VaR Methodologies

There are few VaR methodologies that are very simple and easy to implement, to name a few are (a) Normal (parametric using variance and covariance approach) and (b) Historical simulation. Cleverly these simple methods have been extended with application of weights – recent events are given more weight and past is given less. However, different people have used different weighting methodologies. Riskmetrics has used 'exponentially moving average' where the decay factor (λ) has been considered as 0.94 while Boudoukh, et al. (1997) fixed it at 0.98. We will discuss all these issues shortly and calculate the VaR number and see how they are comparable.

There are also complex methods like EVT and Expected Shortfalls that require higher computing skills but not difficult to implement. EVT has two lines of thought – (a) simpler being the block maxima/minima and generalized extreme value in a Pareto optimality framework and (b) the Hill estimator and modeling both sides of the tail separately.

3.3. Variance-Covariance (Normal) Method

The Variance-Covariance (Normal) method is the easiest of the VaR methodologies. For foex portfolio, the plain standard deviation would be useful to calculate the require VaR. But whether to take static variance of the entire time series or conditional variance is a point for debate. It is argued that variance changes over time horizons and hence we should not rely on unconditional variance for measuring VaR. We will look at both the options.

The normal method assumes normality in the financial time series. In recent past interest in econometrics and empirical finance has revolved around modeling the temporal variation in financial market volatility. Probability distributions for asset returns often exhibit fatter tails than the standard normal, or Gaussian, distribution. The fat tail phenomenon is known as excess kurtosis. Time series that exhibit a fat tail distribution are often commonly referred to as leptokurtic. In addition, financial time series usually exhibit a characteristic known as volatility clustering, in which large changes tend to follow large changes, and small changes tend to follow small changes. In either case, the changes from one period to the next are typically of unpredictable sign. Large disturbances, positive or negative, become part of the information set used to construct the variance forecast of the next period's disturbance. In this manner, large shocks of either sign are allowed to persist, and can influence the volatility forecasts for several periods. Volatility clustering, or persistence, suggests a time-series model in which successive disturbances are serially correlated.

The volatility-clustering phenomenon can be captured through modelling conditional heteroscadasticity, assuming normality of the conditional distribution of return. A useful class of such time series model includes ARCH/GARCH or some of their further generalisation. This class of models not only handle volatility clustering but also accounts to a great extent the fat tail effect (or excess kurtosis) typically observed in financial data. The popular Risk-Metric model (J.P.Morgan, 1996) is a simplified form of heteroskedasticity. The Risk-Metric approach actually model conditional variance as a weighted average of past variance and past returns, where exponential weighting scheme for past returns is used as follows.

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda) r_t^2 = \lambda^t \sigma_0^2 + (1-\lambda)\sum_{k=0}^{t-1} \lambda^k r_{t-k}^2 \quad (5)$$

where σ_t^2 and r_t denote conditional variance and return at time t, respectively; and the parameter λ, known as decay factor, satisfy $0 < \lambda < 1$.

For daily data, the value of the decay parameter in the RiskMetric approach is generally fixed at $\lambda=0.94$ (van den Goorberg and Vlaar, 1999).

3.4. Historical Simulation Method

Historical simulation approach provides some advantages over the normal method, as it is not model based, although it is a statistical measure of potential loss. The main benefit is that it can cope with all portfolios that are either linear or non-linear. The method does not assume any specific form of the distribution of price change/return. The method captures the characteristics of the price change distribution of the portfolio, as VaR is estimated on the basis of actual distribution. This is very important, as the HS method would be on the basis of available past data. If the past data does not contain highly volatile periods, then HS method would not be able to capture the same. Hence, HS should be applied when we have very large data points that are sufficiently large to take into account all possible cyclical events. HS method takes a portfolio at a point of time and then revalues the same using the historical price series. Once we calculate the daily returns of the price series, then sorting the same in an ascending order and find out the required data point at desired percentiles. Linear interpolation can be used if the required percentile falls in between 2 data points. The moot question is what length of price series should be used to compute VaR using HS method and what we should do if the price history is not available. It has to be kept in mind that HS method does not allow for time-varying volatility.

Another variant of HS method is a hybrid approach put forward by Boudhoukh, et al. (1997), that takes into account the exponential declining weights as well as HS by extimating the percentiles of the return directly, using declining weights on past data. As described by Boudhoukh et al. (1997, pp. 3), "the approach starts with ordering the returns over the observation period just like the HS approach. While the HS approach attributes equal weights to each observation in building the conditional empirical distribution, the hybrid approach attributes exponentially declining weights to historical returns". The process is simplified as follows:

- Calculate the return series of past price data of the security or the portfolio from *t-1* to *t.*
- To each most recent K returns: *R(t),* R(*t-1*), ……R(*t-K+1*) assign a weight $[(1-\lambda)/(1-\lambda^k)],[(1-\lambda)/(1-\lambda^k)]\lambda,.....[(1-\lambda)/(1-\lambda^k)]\lambda^{k-1}$ respectively. The constant $[(1-\lambda)/(1-\lambda^k)]$ simply ensures that the weights sum to 1.
- Sort the returns in ascending order.
- In order to obtain *p*% VaR of the portfolio, start from the lowest return and keep accumulating the weights until *p*% is reached. Linear interpolation may be used to achieve exactly *p*% of the distribution.
- In many studies lambda (λ) has been used as 0.98.

3.5. Extreme Value Theory – Hill's Estimator and VaR Estimation

In financial literature, it is widely believed that high frequency return has fatter tails than can be explained by the normal distribution. The tail-index measures the amount of tail fatness of return distribution and fit within the extreme value theory (EVT). One can therefore, estimate the tail-index and measure VaR based on that. The basic premises of this idea stems from the result that the tails of every fat-tailed distribution converge to the tails of Pareto distribution. The upper tail of such a distribution can be modeled as,

$$Prob[X > x] \approx C^{\alpha} \; |x|^{-\alpha} \quad (\text{i.e. } Prob[X \le x] \approx 1 - C^{\alpha} \; |x|^{-\alpha}); \quad x > C \qquad \textbf{..... (6)}$$

Where, C is a threshold above which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

Similarly, lower tail of a fat-tailed distribution can be modeled as

$$Prob[X > x] \approx 1 - C^{\alpha} \; |x|^{-\alpha} \quad (\text{i.e. } Prob[X \le x] \approx C^{\alpha} \; |x|^{-\alpha}); \quad x < C \qquad \textbf{..... (7)}$$

Where, C is a threshold below which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

In practice, observations in upper tail of the return distribution are generally positive and those in lower tail are negative. Thus, both of equation (6) and equation (7) have importance in VaR measurement. The holder of a short financial position suffers a loss when return is positive and therefore, concentrates on upper-tail of return distribution(i.e. equation 6) while calculating his VaR (Tsay, 2002, pp. 258). Similarly, the holder of a long financial position would model the lower-tail of return distribution (i.e. use equation 7) as a negative return makes him suffer a loss.

From equation (6) and (7), it is clear that the estimation of VaR is crucially dependent on the estimation of tail-index α. There are several methods of estimating tail-index and in the present paper, we consider two approaches, viz. (i) Hill's (1975) estimator and (ii) the estimator under ordinary least square (OLS) framework suggested by van den Goorbergh (1999). We consider here the widely used Hill's estimator, a discussion on which is given below.

3.5.1Hill's Estimator

For given threshold C in right-tail, Hill (1975) introduced a maximum likelihood estimator of $\gamma = 1/\alpha$ as

$$\hat{\gamma} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{X_i}{C}\right) \qquad \textbf{..... (8)}$$

where X_i's, i=1,2, …..,n are n observations (exceeding C) from the right-tail of the distribution.

In practice, however, C is unknown and needs to be estimated. If sample observations come from Pareto distribution, then C would be estimated by the minimum observed value, the minimum order statistic. However, here we are not modeling complete portion of Pareto distribution. We are only dealing with a fat-tailed distribution that has right tail that is approximated by the tail of a Pareto distribution. As a consequence, one has to select a threshold level, say C, above which the Pareto law holds. In practice, equation (8) is evaluated based on order statistics in the right-tail and thus, the selection of the order statistics truncation number assumes importance. In other words, one needs to select the number of extreme observations n to operationalise equation (8). Mills (1999, pp. 186) discusses a number of available strategies for selecting n and a useful technique for the purpose is due to Phillips, et al. (1996). This approach makes an optimal choice of n that minimises the MSE of the limiting distribution of $\hat{\gamma}$. To implement this strategy, we need estimates of γ for truncation numbers $n_1 = N^{\delta}$ and $n_2 = N^{\tau}$, where $0 < \delta < 2/3 < \tau < 1$. Let $\hat{\gamma}_j$ be the estimate of γ for $n = n_j$, j=1,2. Then the optimal choice for truncation number is $n = [\lambda T^{2/3}]$, where λ is estimated as $\hat{\lambda} = |(\hat{\gamma}_1/\sqrt{2})(T/n_2)(\hat{\gamma}_1 - \hat{\gamma}_2)|^{2/3}$. Phillips et al. (1996) recommended setting δ =0.6 and τ = 0.9 (see Mills, 1999, pp. 186).

3.5.1 Estimating VaR Using Hill's Estimator

Once tail-index α is estimated, the VaR can be estimated as follows (van den Goorbergh and Vlaar, 1999). Let p and q (p < q) be two tail probabilities and x_p and x_q are corresponding quantiles. Then $p \approx C^{\alpha} (x_p)^{-\alpha}$ and $q \approx C^{\alpha} (x_p)^{-\alpha}$ indicating that $x_p \approx x_q (q/p)^{1/\alpha}$. Assuming that the threshold in the left-tail of the return distribution corresponds to the m-th order statistics (in assending order), the estimate of x_p be

$$\hat{x}_p = R_{(m)} \left(\frac{m}{np} \right)^{\hat{\gamma}} \qquad \text{..... (9)}$$

where $R_{(m)}$ is the m-th order statistics in the assending order of n observations chosen from tail of the underlying distribution; p is the given confidence level for which VaR is being estimated; $\hat{\gamma}$ is the estimate of γ.

The estimate of VaR (with meanings of notations as defined above) would be

$$\hat{V}^{p}_{t+1|t} = - W_t \, \hat{x}_p \,; \; \hat{x}_p \text{ is estimate of quantile of return distribution} \qquad \text{....(10)}$$

or

$$\hat{V}^{p}_{t+1|t} = W_t \, [1 - \exp(\hat{x}_p)]; \; \hat{x}_p \text{ is estimate of quantile of log-return distribution} \qquad \text{....(11)}$$

The methodology described above estimates tail-index and VaR for right tail of a distribution. To estimate the parameters for left tail, we simply multiply the observations by –1 and repeat the calculations.

3.6. **Estimating Multi-Period VaR from one-period VaR**

In practice above methods are used to estimate VaR numbers daily based on one-day holding period returns. However, for computing capital charge, we need the VaR numbers for longer holding period, say 10-days or 30-days. Using the estimates of 1-period VaR, k-period VaR can be estimated by following approximation;

$$\text{VaR}(k) \approx \begin{cases} \left(\sqrt[\alpha]{k}\right) \text{VaR}(1) & \text{if VaR(1) is estimated through tail-index } \alpha \\ \left(\sqrt{k}\right) \text{VaR}(1) & \text{for other VaR Models} \end{cases} \qquad \text{...(12)}$$

3.7. Evaluation of VaR Models - Back Testing

Any method used for VaR estimation need to satisfy the criteria of back testing using the current data set. Suppose we calculate the VaR numbers with probability level 0.01. We can check the accuracy of a VaR model by counting the number of times VaR estimate fails (i.e. actual loss exceeds estimated VaR), say in 100 days. If we want to calculate VaR of a one-day holding period with 99% confidence level, logically, we are allowing 1 failure in 100 days. But if the number is more than 1, then the model is under predicting VaR numbers and if we find less number of failures the model is over predicting. The Basle Committee provides guidelines for imposing penalty leading to higher multiplication factor, when the number of failure is too high. However, no penalty is imposed when the failure occurs with less frequency than the expected number. Thus, selection of VaR model is a very difficult task. A model, which overestimates VaR, may result in reduced number of failure but increase the required capital charge directly. On the other hand if a underestimates VaR numbers, the number failures may be too large which ultimately increases the multiplying factor and hence the required capital charge. Thus an ideal VaR model would be the one, which produces VaR estimates, as minimum as possible and also pass through the backtesting. The BIS requires that models must incorporate past 250 days data points (one year assuming Saturday/Sundays being non-trading days). Accordingly the capital charge is the higher of (i) the previous day's value-at-risk number measured according to the above parameters specified in this section and (ii) the average of the daily value-at-risk measures on each of the preceding sixty business days, multiplied by a multiplication factor prescribed by RBI 3.30 presently for PDs).

To do the back testing, we have used 2 loss functions: (i) an indicator variable *I(t)* which is one if negative return (loss) of the day is more than the VaR for the previous day and zero otherwise. Average of the indicator variable should be our VaR percent; (ii) another loss function where the difference between loss of day *t* and VaR of day *t-1*is squared and multiplied by the indicator variable *I(t)* described above. The higher losses are penalized for a bank. This will make us understand superiority of the model as a model that gives say lower value of the cumulative loss function would be preferred over the others. This is justified in the sense that a model that takes last 500 days of data for back testing should return 5 expected failures at 99% confidence level. But a model that shows only 1 failure but the failure is so huge that it wipes out the capital base because of higher loss intensity while another model may have more than 1 failure but such failures have a low intensity and their cumulative loss function is less than the other case.

3.8. Loss function that addresses the magnitude of the exceptions

However, it may so happen that a Bank might have incurred less number of times loss situations (left tail returns) but the intensity of loss is high that it eats up more capital in comparision to a bank which has faced the situation more number of times but the intensity is very low. Another way of handling the loss function is to assign some values when negative returns (losses) exceeds the daily VaR and loss function should be designed in such a way that it penalizes more for the intensity. This concern can readily be incorporated into a loss function by introducing a magnitude term. Although several are possible, a quadratic term is used here, such that

$$C_{mt+1} = \begin{cases} 1 * (\varepsilon_{t+1} - VaR_{mt})^2 & if loss > Var_{mt} \\ 0 & \end{cases} \quad \ldots(13)$$

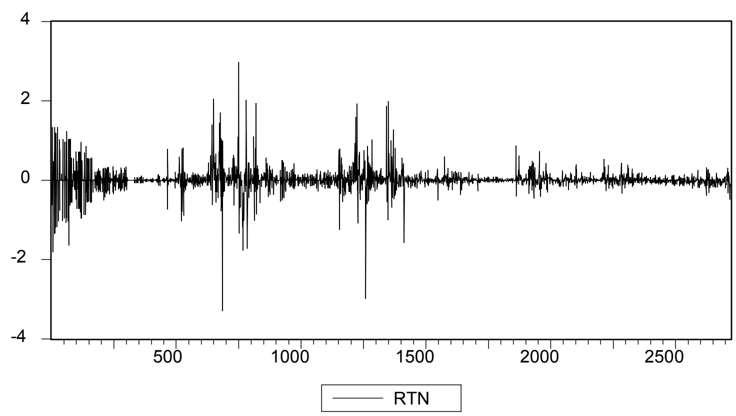
Thus, as before, a score of one is imposed when an exception occurs, but now, an additional term based on the magnitude of the exception is included. The numerical score increases with the magnitude of the exception and can provide additional information on how the underlying VaR model forecasts the lower tail of the underlying f_{t+1} distribution. Unfortunately, the benchmark based on the expected value of C_{t+1} cannot easily be determined because the f_{mt+1} distribution is unknown. However as discussed by Lopez(1998), simple, operational benchmarks based on certain distributional assumptions can be constructed.

4. Data

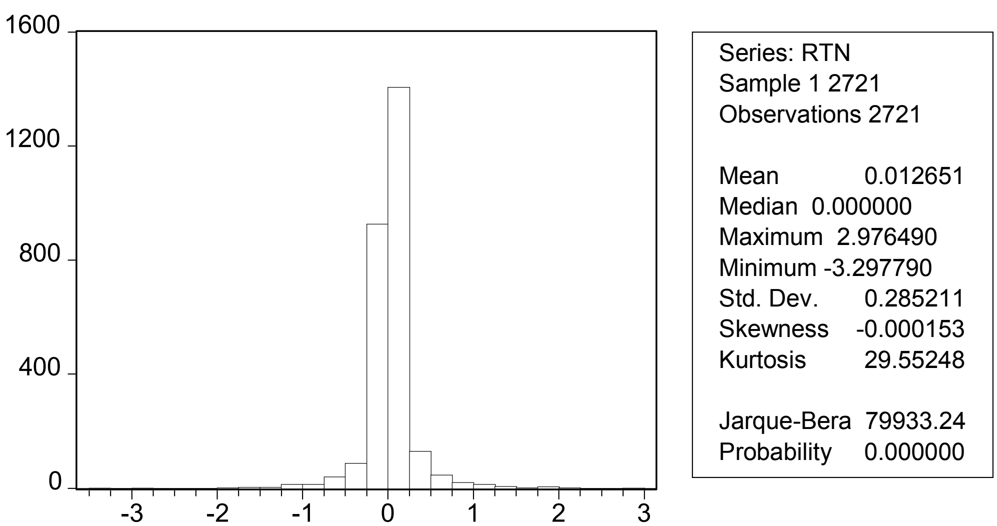
We have used the foreign exchange rates from 01-03-1993 (the date of starting of unified exchange rate system in India) for our analysis and all VaR numbers have been calculated for 08-10-2003. The foreign exchange rates have been collected from RBI and log returns have been calculated for the study.

4.1 Data Characteristics

The return series have spikes at various points in early part of the period under analysis. This can be seen in the *Figure-1*.



The descriptive statistics are given as:



4.2. Empirical Results

In this section we report our estimated VaR figures and corresponding capital charges. All calculations are restricted to left-tail (one tailed) of return distribution and probability level is fixed at 0.01 (equivalently confidence level of VaR estimates is set to 0.99). Thus, the estimates we provide here actually refer to long-investment positions on portfolio containing forex. We have used a 500 days data-rolling window (for about 2 years) for our analysis and back testing and all VaR number have been calculated for October 8, 2003.

We first compute 1-day VaR numbers for all methods and as well as the average of 1-day VaRs in last 60 days in our sample. All VaR estimates correspond to the probability level 0.01 (equivalently correspond to the confidence level 0.99). For a given security/portfolio, maximum of these two VaRs (i.e. 1-day VaR in last day and 60-day average of 1-day VaR) has been adjusted to arrive at VaR numbers corresponding to two alternative holding periods, viz., 10-days and 30-days. For calculating capital charge corresponding to a holding period h, h=10-days or 30-days, the VaR with h-days holding period has been multiplied by the multiplication factor 3.3 (as given in the RBI circular for PDs). Relevant results are given in *Table 1*.

An important issue need to be mentioned here is that all VaR estimates provided are in percentage form, and thus, may actually be termed as the relative VaR (Wong, et al., 2003), which refers to the percentage of a portfolio value which may be lost after h-holding period with a specified probability (i.e. the probability level of VaR). The absolute VaR (i.e. the VaR expressed in Rupees term) can easily be computed by multiplying the portfolio values with the estimated relative VaR. Similarly, the capital charge can also be represented in two alternative forms, viz., relative (i.e. in percentage) or absolute (i.e. in rupees terms). The additional information we require to convert a relative VaR/capital charge in a day to a corresponding absolute term (i.e. rupees term) figures is the value of the portfolio/security.

Table 1: Estimated VaRs and Capital Charges

Portfolio	Description of Estimate	Variance-Covariance (Normal) Method					Historical Simulation		Tail-Index (Hill's Estimator)	
		Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)						
		Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
Fx	Last day VaR	0.6772	0.2330	0.3426	0.4000	0.4417	0.9554	0.3195	0.9121	0. 2935
	60-Day Avg	0.6796	0.2269	0.2672	0.2675	0.2617	0.9566	0.2626	0.9135	0.2606

Max (Last day, 60-Day Avg)	0.6796	0.2330	0.3426	0.4000	0.4417	0.9566	0.3195			0.9135	0.2935
10-Day Var (%)	2.1490	0.7369	1.0834	1.2648	1.3966	3.0250	1.0102	2.8887	0.9281		
30-day Var (%)	3.7221	1.2764	1.8765	2.1907	2.4190	5.2394	1.7497	5.0034	1.6076		
Cap Charge, H=10-day (%)	7.0915	2.4318	3.5752	4.1739	4.6089	9.9824	3.3337	9.5328	3.0628		
Cap Charge, H=30-day (%)	12.283	4.2120	6.1925	7.2294	7.9828	17.2899	5.7741	16.5114	5.3050		

The columns in Table are self-explanatory. As can be seen therein, we estimated VaRs and capital charges for five alternative schemes under normal method, one for full sample estimate, one for rolling sample estimate, and three for Risk Metric approach corresponding to three alternative decay factors, $\lambda = 0.98, 0.96$ and 0.94. Full sample estimates at any day, say t, are derived based on all returns from day 1 to t. In the case of rolling sample estimates, we fix the size/length of the rolling windows at 1584 days. The columns with titles 'Full' and 'Rolling' provide estimates corresponding to full sample and rolling sample, respectively. As regards to historical simulation, we provide both 'full sample' and 'rolling sample' estimates. Same is the case for tail-index (Hill's estimator) approach.

4.3 Back Testing for Competing VaR Models

For evaluating performance of competing VaR models, back testing has been carried out with the daily returns for last 290 days (covering about a period of one year as backtesting observations. Both 'full sample' and 'rolling sample' estimates of VaRs are assessed. The backtesting strategy adopted for the case of rolling sample estimates, is as follows; estimate 1-day VaR using returns for days 1 to 2221 and compare the same with the return of the 2222-th day, estimate 1-day VaR based on returns on days 2 to 2222 and compare the same with 2223-th day's return, and so on. In the case of full sample estimates, VaRs at any day, say t, are estimated based on returns for the days 1 to t.

Table 3 gives the back testing results with failure. As we have used 500 data points for our analysis, the expected failures are 5 (with 99% confidence interval).

Table 3: Results of Back Testing for Two Portfolios

Fx	Variance-Covariance (Normal) Method					Historical Simulation		Tail-Index (Hill's Estimator)	
	Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)						
	Full	Rolling	0.98	0.96	0.94	Full	Rolling	Full	Rolling
	0	10	15	11	8	0	11	0	9

Note: Number in each cell indicates the number of days (out of 250 backtesting days) when actual loss exceeds the VaR (with probability level 0.01). For a good VaR model, this number would be close to 3.

The loss function estimated is given in Table 4.

Table 4: Loss Function Estimation

Variance-Covariance (Normal) Method					Historical Simulation	
Simple (homoscadastic)		Risk Metric with λ (conditional heteroscadasctic)				
Full	Rolling	0.98	0.96	0.94	Full	Rolling
0	3.1254	4.4613	3.3555	2.9836	0	3.7943

Conclusion: Value-at-Risk (VaR) has been widely promoted by the Bank for International Settlement (BIS) as well as central banks of all countries as a way of monitoring and managing market risk and as a basis for setting regulatory minimum capital standards This paper has experimented with a number of available VaR models, such as, variance-covariance/normal (including Risk-Metric approach), historical simulation and tail-index based method for estimating VaR for foreign exchange portfolio.

Empirical results are quite interesting. We found that all the models have been failing in back test except when we take the whole period. When we use a rolling period of 500 days, the models have underestimated the risk. The full sample over-estimates the risk as we did not find a single failure in back testing.

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