STATISTICAL TECHNIQUES TO HANDLE RISK IN CAPITAL BUDGETING

OBJECTIVES

After studying this lesson, you should be able to :

- Explain the use cf Probability technique in handling risk .
- Explain the concepts of standard deviation and coefficient of variation in measuring risk.
- Explain the steps involved in construction of decision tree.

STRUCTURE

6.1 INTRODUCTION

In the previous lesson you have studied the meaning and source of risk . In addition to this conventional techniques to handle risk in capital expenditure projects are also examined . The purpose of this lesson is to explain the use of statistical techniques such as probability ,probability distribution standard deviation , coefficient of variation and decision tree in handling risk in capital budgeting projects.

6.2 PROBABILITY ASSIGNMENT

The concept of probability is one of the statistical techniques to handle risk in capital budgeting projects . It may be described as a measure of sometimes opinion about the likelihood that an event will occur . If an event is certain to occur , we say that it has a probability of occurring in zero. Thus, probability of all events to occur lies between 0 and 1.

A probability distribution may consist of number of estimates. But in simple form it may consist of a few estimates . One commonly used form are " high, low and best guess" estimates, or – the optimistic, most likely and most pessimistic" estimates. For example . the annual cash flows expected from a project could be:

Assumption	Annual Cash Flows (Rs)
Best Guess	1,00,000
High Guess	85,000
Low Guess	40,000

It can be seen that it is an improvement over the single forecast. But this is not sufficient. The forecast should describe more accurately his degree of confidence in his forecasts. i.e. he should describe his feelings as the probability of these estimates occurring. For example, he may assign the following probabilities to his estimates:

Assumption	Annual Cash Flow	Probability
Best Guess	1,00,000	.20
High Guess	85,000	.60
Low Guess	40,000	.20

Once the probability assignments are given to the future cash flows the next step is to find the expected net present value . The net present value can be found by multiplying the monetary values of the cash flows by their probabilities and discount rate . Consider the above example (assume investment as 60,000)

Assumption	Annual cash flow (Rs)	Probability	Expected cash flow
Best Guess	1,00,000	.20	20,000
High Guess	85,000	.60	51,000
Low Guess	40,000	.20	8,000
		Expected cash flow	79,000

ILLUSTRATION 6.1

X Co. Ltd., has given the following possible cash inflows of two of their projects X and Y. Both of their projects will require an equal investment of Rs. 5,000/- You are required to suggest which project should be accepted 10% discount rate.

Possible event	Project X		Project Y	
	Cash Inflow	Probability	Cash Inflow	Probability
A	4,000	.10	12,000	.10
B	5,000	.20	10,000	.15
C	6,000	.40	8,000	.50
D	7,000	.20	6,000	.15
E	8,000	.10	4,000	.10

Solution :

Calculation of expected cash flows for Project X and Project Y .

Event	Cash Inflows (Rs)	Project X		Project Y		
		Probability	Expected cash flow	Cash flow	Probability	Expected cash flow
A	4,000	.10	400	12,000	.10	1,200
B	5,000	.20	1,000	10,000	.15	1,500
C	6,000	.40	2,400	8,000	.50	4,000
D	7,000	.20	1,400	6,000	.15	900
E	8,000	.10	800	4,000	.10	400
	Total		6,000		Total	8,000

The above calculations show that project Y has higher expected cash flow as compared to project X. If expected cash/flow are discounted @ 10 % the net present value for project X will be : Rs. (6,000 x .909) – 5,000/- = Rs 454/-. The net present value of project Y will be (8000 x .909) – 5000 = Rs. 2272/-. Form the calculations it can be seen that NPV of project Y is more than project X. It is advisable to accept project Y.

6.3 PROBABILITY DISTRIBUTION APPROACH

In the previous section , we had introduced the use of the concept of probability for incorporating risk in evaluating capital budgeting proposals. As already observed, the probability distribution of cash flows over time provides valuable information about the expected value of return and the dispersion of the probability distribution of possible returns. On the basis of this information an accept-reject decision can be taken. We discuss the application of probability theory to capital budgeting in this section.

The application of this theory in analysing risk in capital budgeting depends upon the behaviour of the cash flows, from the point of view of behavioural cash flows being (i) independent, or (ii) dependent. The assumption that cash flows are independent over time signifies that future cash flows are not affected by the cash flows in the preceding or following years. Thus, cash flows in year 3 are not dependent on cash flows in year 2 and so on. When cash flows in one period depend upon the cash flows in previous periods, they are referred to as dependent cash flows.

Independent Cash Flows Over Time: The mathematical formulation to determine the expected values of the probability of NPV for any project is:

$$NPV = \sum_{t=1}^{n} \frac{\overline{CF_t}}{(1+i)^t} - CO$$

where CF_t is the expected value of net CFAT in period t and is the riskless rate of interest. The standard deviation of the probability distributed of NPV is equal to

$$\sigma NPV = \sqrt{\sum_{t=1}^{n} \frac{\sigma_t^2}{(1+i)^{2t}}}$$

Where s_t is the standard deviation of the probability distribution of expected cash flows for period t, s, would be calculated as follows:

$$\sigma t = \sqrt{\sum_{t=1}^{m} (CF_{jt} - \overline{CF_t})^2 . P_{jt}}$$

The above calculations of the standard deviation and the NPV will produce significnat volume of information for evaluating the risk of the investment proposal. The calculations are illustrated in Example 6.2.

Illustration 6.5

Suppose there is a project which involves initial cost of Rs. 20,000/- (cost at t = 0). It is expected to generate net cash flows during the first 3 years with probability as shown below.

Expected Cash Flows

Year 1		Year 2		Year 3	
Probability	Net Cash Flows	Probability	Net Cash Flows	Probability	Net Cash Flows
0.10	Rs. 6,000	0.10	Rs. 4,000	0.10	Rs. 2,000
0.25	8,000	0.25	6,000	0.25	4,000
0.30	10,000	0.30	8,000	0.30	6,000
0.25	12,000	0.25	10,000	0.25	8,000
0.10	14,000	0.10	12,000	0.10	10,000

Solution

(i) **Expected Value :** For the calculation of standard deviation for different periods, the expected values are to be calculated first. These are calculated as follows.

Calculation of Expected Values of Each Period

	Probability (1)	Net cash flow (2)	Expected value (1x2) (3)
Year 1	0.10	Rs. 6,000	Rs. 600
	0.25	8,000	2,000
	0.30	10,000	3,000
	0.25	12,000	3,000
	0.10	14,000	1,400
			CF_1 = 10,000
Year 2	0.10	4,000	400
	0.25	6,000	1,500
	0.30	8,000	2,400
	0.25	10,000	2,500
	0.10	12,000	1,200
			CF_2 = 8,000
Year 3	0.10	2,000	200
	0.25	4,000	1,000
	0.30	6,000	1,800
	0.25	8,000	2,000
	0.10	10,000	1,000
			CF_3 = 6,000

(ii) The standard deviation of possible net cash flows is;

$$\sigma t = \sqrt{\sum_{t=1}^{m} (CF_{jt} - \overline{CF_t})^2 . P_{jt}}$$

thus, the standard deviation for period 1 is :

$$\sigma_1 = \sqrt{\begin{array}{l}[0.10(6000-10,000)^2 + 0.25(8,000-10,000)^2 + 0.30(10,000-10,000)^2 \\ +0.25(12,000-10,000)^2 + 0.10(14,000-10,1000)^2\end{array}}$$

$= \text{Rs. } 2,280$

When calculated on similar lines the standard deviations for periods 2 and 3 (s_2 and s_3) also work out to Rs. 2,280.

iii) NPV = Rs. 10,000 (0.909) + Rs. 8,000 (0.826) + Rs. 6,000 (0.751) – Rs. 20,000 = Rs. 204.

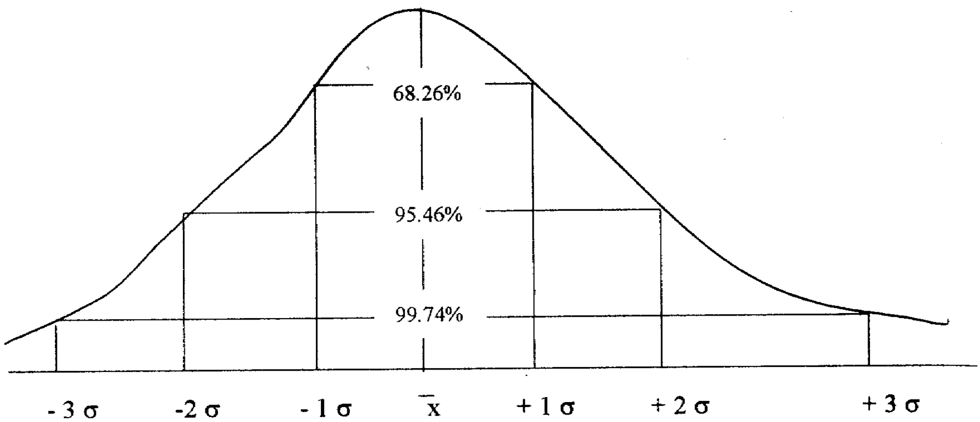
(iv) The standard deviation under the assumption of independence of cash flow over time:

$$\sigma NPV = \sqrt{\sum_{t=1}^{n} \frac{\sigma_t^2}{(1+i)^{2t}}} = \sqrt{\frac{Rs.\ (2,280)^2}{(1.10)^2} + \frac{Rs.\ (2,280)^2}{(1.10)^4} + \frac{Rs.\ (2,280)^2}{(1.10)^6}}$$

$= \text{Rs. } 3,283$

Normal Probability Distribution : We can make use of the normal probability distribution to further analyze the element of risk in capital budgeting. The use of the normal probability distribution will enable the decision maker to have an idea of the probability of different expected values of NPV, that is, the probability of NPV having the value of zero or less; greater than zero and within the range of two values, say, Rs. 1,000 and Rs. 1,500 and so on. If the probability of having NPV of zero or less is considerably low, say, .01, it implies that the risk in the project is negligible. Thus, the normal probability distribution is an important statistical technique in the hands of decision-makers for evaluating the riskiness of a project.

The normal probability distribution as shown below has a number of useful properties.



The area under the normal curve, representing the normal probability distribution, is equal to 1 (0.5 on either side of the mean). The curve has its maximum height at its expected value (mean). The distribution (curve) theoretically runs from minus infinity to plus infinity. The probability of occurrence beyond 3ss is very near zero (0.26 per cent).

For any normal distribution, the probability of an outcome falling within plus or minus 1 from the mean is 0.6826 or 68.26 per cent. If we take the range within 2, the probability of an occurrence within this range is 95.46 and 99.74 per cent of all outcomes and lie within 3ss of the x.

Illustration 6.3

Assume that a project has a mean of Rs. 40 and standard deviation of Rs. 20. The management wants to determine the probability of the NPV under the following ranges:

i) Zero or less,

ii) Greater then zero,

iii) Between the range of Rs. 25 and rs. 45,

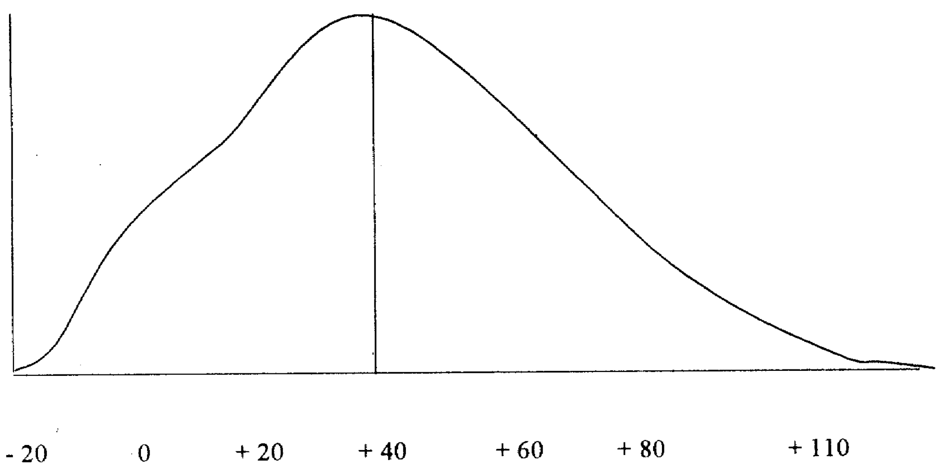
iv) Between the range of Rs. 15 and Rs. 30.

Solution

i) **Zero or Less :** The first step is to determine the difference between the expected outcome X and the expected net present value x . The second step is to standardise the difference (as obtained in the first step) by the standard deviation of the possible net present values. Then, the resultant quotient is to be seen in statistical tables of the area under the normal curve. Such a table (Table Z) is given at the end of the book. The table contains values for various standard normal distribution functions. Z is the value which we obtain through the first two steps, that is:

$$Z = \frac{0 - \text{Rs. } 40}{\text{Rs. } 20} = -2.0$$

This is also illustrated in shown in the following Figure.



The above figures indicates that a NPV of 0 lies 2 standard deviation to the left of the expected value of the probability distribution of possible NPV. Table Z indicates that the value within the range of 0 to 40 is 0.4772. Since the area of the left-hand side of the normal curve is equal to 0.5, the probability of NPV being zero or less would be 0.0228, that is, 0.5 – 0.4772. it means that there is 2.28 per cent probability that the NPV of the project will be zero or less.

ii) **Greater than zero :** The probability for the NPV being greater than zero would be equal to 97.72 per cent, that is, 100 – 2.28 per cent probability of NPV being zero or less.

iii) **Between the range of Rs. 25 And Rs. 45 :** The first step is to calculate the value of Z of the two ranges : (a) Between Rs 25 and Rs 40. And (b) Between Rs 40 and Rs 45. The second and the last step is to sum up the probability obtained from these values .

$$Z_1 = \frac{Rs.\ 25 - Rs.\ 40}{Rs.\ 20} = -0.75$$

$$Z_2 = \frac{Rs.\ 45 - Rs.\ 40}{Rs.\ 20} = +0.25$$

The area as per Table Z for the respective values of – 0.75 and 0.25 is 0.2734 and 0.0987 respectively . Summing up we have 0.3721. In other words there is 37.21 per cent probability NPV being within the range of Rs 25 and Rs 45 .(It may be noted that the negative signs for the value of Z in any way does not effect the way Table Z is to be consulted . It simply reflects that the values lies to the left of the mean value)

(iv) Between the range of Rs 15 and Rs 30.

$$Z_1 = \frac{Rs.\ 15 - Rs.\ 40}{Rs.\ 20} = -1.25$$

$$Z_2 = \frac{Rs.\ 30 - Rs.\ 40}{Rs.\ 20} = -0.50$$

According to Table Z, the area for the respective values –1.25 and –0.5 is 0.3944 and 0.1915. . The probability of having value between Rs 15 and 40 is 39.44 per cent , while the probability of having value between Rs 30 and 40 = 19.15 percent. Therefore, the probability of having value between Rs 15 and Rs 30 would be 20.29 per cent = (39.44 per cent – 19.15 per cent)

6.4 STANDARD DEVIATION

The probability assignment approach to risk analysis in capital budgeting does not provide the decision maker about the variability of cash flows and therefore the risk. To overcome this limitation standard deviation technique is used. Standard deviation is an absolute measure of risk. It may be defined as a square root of squared deviations calculated from the mean . In case of capital budgeting this measure is used to compare the possible variability of cash flows of different projects from their respective mean. A project having larger standard deviation will be more risky as compared to a project having smaller standard deviation.

The following steps are involved in calculating standard deviation:

1. Mean value of possible cash flow is computed.
2. Deviation between the mean value and the possible cash flows are found out.
3. Deviation is squared.

4. Squared deviations are multiplied by the assigned possibilities which give the weighted squared deviation.

5. The weighted squared deviations are totalled and their square root is found out. The resulting figure is standard deviation.

Standard deviation is calculated by using the following formula.

Where n = No. of years

R = Expected cash Flows

R = Mean value of cash flows

P = Probability assignments

Illustration 6.4

Consider the data given in illustration 1 and calculate standard deviation.

Solution :

Project X

Events	**Cash Inflows (R)**	**(R – R) R = 6000**	**(R – R)2**	**Pi**	**(R – R)2 Pi**
A	4000	2000	4000000	.10	400000
B	5000	1000	1000000	.20	200000
C	6000	0	0	.40	0
D	7000	1000	1000000	.20	200000
E	8000	2000	4000000	10	400000
ER	3000				1200000

$$R = \frac{\Sigma R}{N} = \frac{3000}{5} = 6000 \quad \sigma_x = \sqrt{1200000}$$

$$\sigma_x = \text{Rs. } 1095$$

Project Y

Events	Cash Inflows (R)	(R – R) R = 8000	(R – R)2	Pi	(R – R)2 Pi
A	12000	-4000	16000000	.10	1600000
B	10000	-2000	4000000	.20	600000
C	8000	0	0	.40	0
D	6000	2000	1000000	.20	600000
E	4000	4000	16000000	.10	1600000
ER	3000				4400000

$$R = \frac{\Sigma R}{N} = \frac{40000}{5} = 8000 \quad \sigma_y = \sqrt{4400000}$$

$$\sigma_y = \text{Rs. } 2098$$

The standard deviation of project X is Rs. 1,095 as of Project Y Is Rs 2,098. Thus variability of cash flow is more in case of Project Y as compared to project X. Hence, Project Y is more risky.

6.5 COEFFICIENT OF VARIATION

Coefficient of variation is a relative measure of risk. It is defined as a standard deviation of probability distribution devided by its expected value . It is calculated as follows

$$\text{Coefficient of variation} = \frac{\text{Standard Deviation}}{\text{Expected(Mean)Value}}$$

Consider the illustration 6.4

Project X $\frac{1095}{6000} = 1.825$

Project Y $\frac{2098}{8000} = 0.2623$

The coefficient of variation of Project Y is more as compared to Project X. Hence Project Y is more risky. Whether Project X or Project Y should be accepted will depend upon the investors attitude towards risk. He would prefer Project Y if he is ready to bear more risk in order to get higher monitory value. In case if he has great aversion to risk, he would accept Project X as it is less risky.

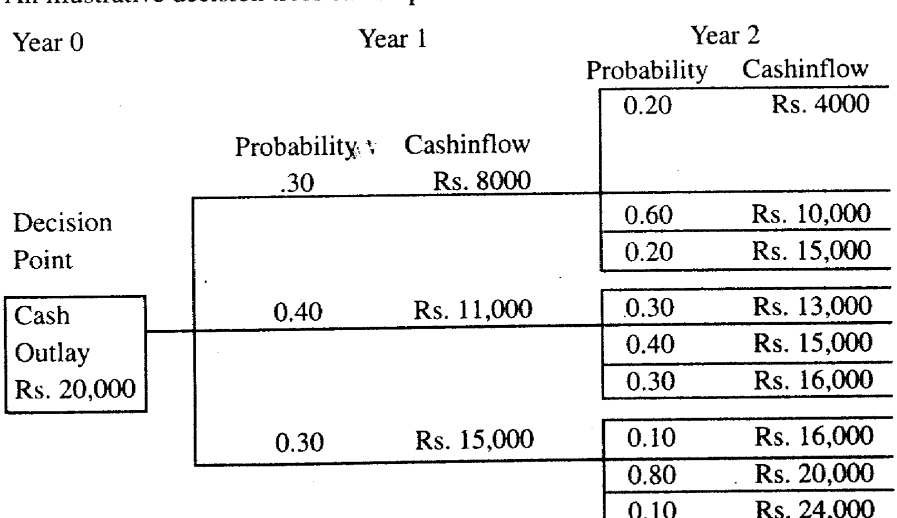
6.6 DECISION TREE ANALYSIS

Decision Tree Analysis is another technique of analyzing the risk involved in capital budgeting proposals. Decision Tree is a graphic display of relationship between a present decision and possible future events, future decision and their consequences . The sequence of events is mapped out over time in a format similar to the branches of a tree. In otherwords, it is a pictorial representation in tree form which indicates the magnitude , probability and interrelationships of all possible outcomes.

The following steps are taken for constructing a decision tree .

1. **Definition of the proposal :** The first step in constructing a decision tree is to define the proposal .For example , entering a new market or introducing new product line .
2. **Identify decision alternatives :** The decision alternatives should be clearly identified. For example a firm may be considering , the purchase of a new plant for manufacturing a new product. It may have 3 alternatives (a) Purchase a small plant (b) Purchase a large plant (c) Purchase a medium size plant.
3. **Draw a decision tree :** The decision tree is then laid down showing decision point and decision branches.
4. **Analysis of data :** The results should be analyzed and the best alternative should be selected.

An illustrative decision trees can be presented as follows:



Decision Tree

The technique of a decision tree analysis has the advantage of giving an overall view of all possibilities associated associated with the project. The management can take a decision taking the entire picture in mind. However it has one big disadvantage. Its format may become unwisely and complex if the project has a long life.

6.7 SUMMARY

In order to analyze the risk involved in capital budgeting statistical techniques can be used. Statistical techniques include probability, probability distribution assignments standard deviation, coefficient of variation and decision tree analysis.

6.8 SELF EXAMINATION QUESTIONS

1. Explain the use of probability assignments in estimating the risk involved in capital investment proposals.
2. Standard deviation is an absolute measure of risk explain?
3. What is coefficient of variation and how do you compute.
4. Define decision tree. What are the steps involved in constructing decision tree
5. Explain the statistical techniques to handle risk in capital budgeting projects.

6.9 A company is considering an investment in a project that requires an initial net investment of Rs 3,000 with an expected cash flow (CFAT) generated over three years as follows.

Year 1		Year 2		Year 3	
CFAT Rs.	**Probability**	**CFAT Rs.**	**Probability**	**CFAT Rs.**	**Probability**
800	0.10	800	0.10	800	0.20
1000	0.20	1000	0.30	1000	0.50
1500	0.40	1500	0.40	1500	0.20
2000	0.30	2000	0.20	2000	0.10

a) What is the expected cash flows of each project?

b) Calculate NPV assuming risk free rotate of 5 %?

c) Calculate the standard deviation about the expected value ?

d) Find the probability that the NPV will be less than Zero(Assume that the distribution is natural and continuous).

e) What is the probability that NPV will be greater than zero?

f) What is the probability that NPV will be (I) between the range of Rs 500 and Rs. 750 (iii) between the range of Rs 400 and Rs 600 (iii) at least Rs 300 and (iv) at least Rs 1000 ?

ANSWERS:

a) Year I – Rs 1480, Year 2- Rs 1380, Year 3- Rs 1160.

b) Rs 663

c) σ_1Rs. 417, σ_2 Rs. 400, σ_3 Rs 362, σ Rs. 622.

d) 14.23 per cent e) 85.77 per cent

f) i) 15.83 per cent ii) 7.19 per cent iii) 29.46 per cent .

6.10 KEY WORDS

Probability - It is someone's opinion about the likelihood that an event will occur.

Standard Deviation – It is the square root of standard deviations calculated from the mean .

Coefficient of Variation - It is the product of standard deviation of probability distribution divided by its expected value(Mean)

6.10 BOOK FOR FURTHER READING

1. Prasannes Chandra, Financial Management, Tata MC-Graw Hill Company Co. Ltd., New Delhi, 5th ed 2002
2. I.M. Pandey, Financial Management, Vikas Publishing House, New Delhi, 8th ed. 2000
3. M.Y. Khan & P.K. Jain, Financial Management, Tata Mc-Graw Hill Co.Ltd, New Delhi, 3rd ed, 2002
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