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Drawdown at a large-diameter observation well

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ABSTRACT

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A generalized discrete kernel approach has been used to analyse the effect of both production well storage and observation well storage on drawdown at any point in the aquifer during pumping and recovery.

Non-dimensional time–drawdown plots have been presented for four different combinations of a production well and an observation well, which may or may not have storage. The non-dimensional time–drawdown plots include the response of the aquifer during the recovery phase. The contribution from the observation well storage to the aquifer during pumping, and the replenishment of the observation well storage during recovery have been presented for specific cases.

NOMENCLATURE

$E_1(X)$	an exponential integral defined as $E_1(X) = \int_X^\infty \frac{e^{-y}}{y} dy$
M, m, n	positive integers
$Q_A(n)$	quantity of water withdrawn from aquifer storage at the production well during nth time step
$Q_O(n)$	quantity of recharge taking place from the observation well storage during nth time step
$Q_p(n)$	quantity of water pumped from the well during nth time step
$Q_W(n)$	quantity of water withdrawn from production well storage during nth time step
r	distance measured from the centre of the production well to a specific point in the aquifer
r_{co}	radius of the observation well casing
r_{cp}	radius of the production well casing
r_{wo}	radius of the observation well screen

r_{wp}	radius of the production well screen
r_1	distance between production well and observation well
r_2	distance between the point under consideration and the centre of the observation well
$s_{ao}(n)$	drawdown of the piezometric surface in the aquifer at the observation well face at the end of nth time step
$s_{ap}(n)$	drawdown of the piezometric surface in the aquifer at the production well face at the end of nth time step
$s_{wo}(n)$	drawdown of the water surface in the observation well due to recharge from observation well storage to the aquifer at the end of nth time step
$s_{wp}(n)$	drawdown of the water surface at production well due to abstraction from production well storage at the end of nth time step
S	storage coefficient of the aquifer
t	time since pumping commenced
t_p	total time of pumping
T	transmissivity of the aquifer
α_o	storage parameter of observation well $= S(r_{wo}/r_{co})^2$
α_p	storage parameter of production well $= S(r_{wp}/r_{cp})^2$
$\delta_r(M)$	$\frac{1}{4\pi T}\left\{E_1\left(\frac{Sr^2}{4TM}\right) - E_1\left[\frac{Sr^2}{4T(M-1)}\right]\right\}$
$\delta_{r_{wo}}(M)$	$\frac{1}{4\pi T}\left\{E_1\left(\frac{Sr_{wo}^2}{4TM}\right) - E_1\left[\frac{Sr_{wo}^2}{4T(M-1)}\right]\right\}$
$\delta_{r_{wp}}(M)$	$\frac{1}{4\pi T}\left\{E_1\left(\frac{Sr_{wp}^2}{4TM}\right) - E_1\left[\frac{Sr_{wp}^2}{4T(M-1)}\right]\right\}$
$\delta_{r_1}(M)$	$\frac{1}{4\pi T}\left\{E_1\left(\frac{Sr_1^2}{4TM}\right) - E_1\left[\frac{Sr_1^2}{4T(M-1)}\right]\right\}$
$\delta_{r_2}(M)$	$\frac{1}{4\pi T}\left\{E_1\left(\frac{Sr_2^2}{4TM}\right) - E_1\left[\frac{Sr_2^2}{4T(M-1)}\right]\right\}$

INTRODUCTION

An aquifer test can be conducted in a large-diameter well. In such a case the aquifer response may be recorded either in the large-diameter well itself or at a nearby observation well of negligible diameter. A large-diameter well can also serve as an observation well if an aquifer test is conducted in a production well of negligible diameter. The storage associated with a large-diameter production well or observation well modifies and causes delay to the aquifer response. Therefore, storage effect should be duly considered when solving a direct or an inverse problem.

Papadopulos and Cooper (1967) solved the problem of unsteady flow to a large-diameter production well in a confined aquifer. Using their solution, the aquifer response can be estimated at the production well and at other observation wells which have negligible storage. Barker (1984) has shown that, if a pumping test is conducted in a production well of negligible diameter, the drawdown in a large-diameter observation well is identical to the drawdown in an observation well if the roles of the wells are reversed. Mucha and Paulikova (1986) suggested a method for accounting for the well storages in the computation of drawdown at any point in a confined aquifer. The method makes use of unit step response function coefficients and convolution technique. In this method the contributions of the production well storage and the observation well storage at different times are computed from respective observed drawdown values at the wells. Fenske (1977) derived a set of equations based on the Theis' solution for finding the aquifer response when both the observation well and the production well have storage. In order to account for the effect of storage in the observation well, Fenske assumed that the water stored in the observation well recharges the aquifer instantaneously with a drop in head in the aquifer. Barker (1984) observed that if both the production well and the observation well have storage, the solution for the drawdown is unknown. In the present paper, a generalized discrete kernel approach is described to analyse the effect of the production well and the observation well storage on drawdown at any point in the aquifer.

The discrete kernel method presented here is approximate because the discrete kernel coefficients are generated making use of the Theis' solution, which is based on the assumption that the well is infinitesimally narrow. An exact solution to the problem of unsteady flow to a well of finite radius for uniform withdrawal from aquifer storage is yet to be found. An asymptotic solution was obtained by Hantush (1964) according to which the Theis' formula is valid for any value of the well screen radius, r_{wp}, at a non-dimensional time parameter, $4Tt/(Sr_{wp}^2)$, greater than 120. This limitation should be considered when analysing a large-diameter well problem by the discrete kernel approach. A solution to the problem of unsteady flow to a large-diameter well by the discrete kernel approach has already been compared with the exact solution given by Papadopulos and Cooper (1967), and the approximate discrete kernel method is found to compare well with the exact solution (Patel and Mishra, 1983).

STATEMENT OF THE PROBLEM

In an aquifer test the drawdowns are generally recorded at several observation wells or piezometers in addition to the pumping well. The number of observation points may be restricted for rapid exploration or for economy. If the aquifer is homogeneous, a single observation well can serve the purpose of solving the inverse problem. Let there be only one observation well located at a distance r_1 from the pumping well. The pumping well and the observation well may have significant storage depending upon their radii. In an aquifer test conducted with a single observation well, one of the four cases shown in Fig. 1 is met. Let the radii of the screened and unscreened parts of the production well be r_{wp} and r_{cp}, respectively, and those of the observation well be r_{wo} and r_{co}. Let the confined aquifer be homogeneous, isotropic, infinite in lateral extent, and initially at rest condition. Let the pumping be continued up to time t_p. The rate of pumping may be constant or it may vary with time. It is necessary to determine the drawdown in the piezometric surface at the large-diameter observation well, at the production well, and at any distance 'r' from the centre of the production well during pumping as well as during the recovery period.

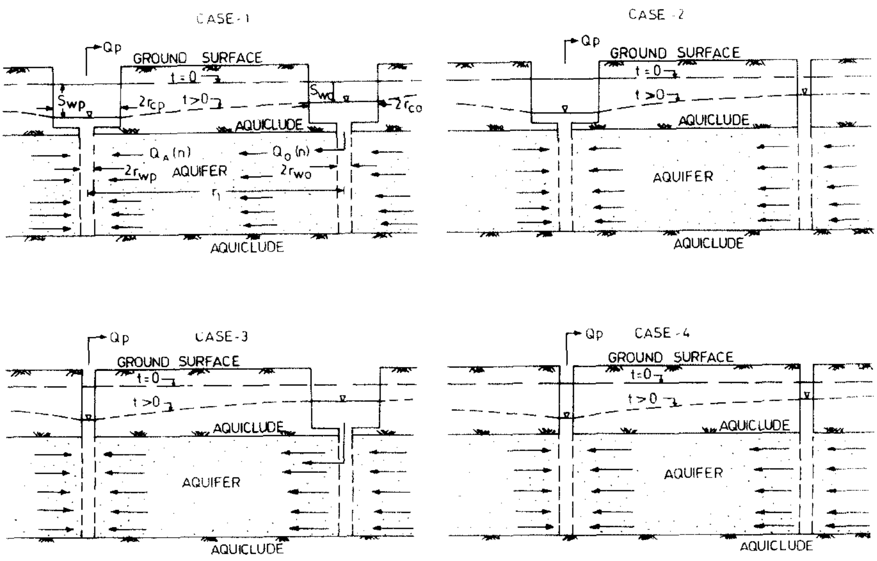

Fig. 1. Schematic diagrams of a production well and observation well with or without storage.

ANALYSIS

The following assumptions have been made in the analysis:

(1) The radii of the production well screen and the observation well screen are small.

(2) The time parameter is discrete. Within each time step the abstraction rate from the well storage and that derived from the aquifer storage are separate constants.

(3) At any time, the drawdown of the piezometric surface in the aquifer at the well face is equal to the drawdown of the water level in the well. This assumption is true for both the production well and the observation well.

The basic differential equation for an axially symmetric radial unsteady groundwater flow in a homogeneous, isotropic, confined aquifer of uniform thickness is given by

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \tag{1}$$

where s is the drawdown, r is the distance measured from the centre of the well, t is the time, S is the storage coefficient and T is the transmissivity of the aquifer.

For the initial condition $s(r, 0) = 0$, and the boundary condition $s(\infty, t) = 0$, the solution to the above differential equation, when a unit impulse quantity of water is withdrawn from the aquifer storage through a well with negligible radius, is given by (Muskat, 1937; Carslaw and Jaeger, 1959)

$$s(r, t) = \frac{\mathrm{e}^{-Sr^2/(4Tt)}}{4\pi Tt} \tag{2}$$

The response of the aquifer to a unit impulse excitation has been defined as the unit impulse kernel (Morel-Seytoux, 1975). Designating the unit impulse kernel for drawdown as $k(t)$, the drawdown caused by variable abstraction can be found using the expression (Muskat, 1937; Carslaw and Jaeger, 1959)

$$s(r, t) = \int_0^t Q_A(\tau)k(t - \tau)\,\mathrm{d}\tau \tag{3}$$

where $Q_A(\tau)$ is the variable abstraction rate from the aquifer storage at time τ.

Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each time step but varies from step to step, drawdown at the end of nth time step can be written as (Morel-Seytoux, 1975)

$$s(r, n) = \sum_{\gamma=1}^{n} \delta_r(n - \gamma + 1)Q_A(\gamma) \tag{4}$$

in which the discrete kernel coefficients $\delta_r(M)$ are given by

$$\delta_r(M) = \int_0^1 k(M - \tau)\,\mathrm{d}\tau = \frac{1}{4\pi T}\left\{E_1\left(\frac{Sr^2}{4TM}\right) - E_1\left[\frac{Sr^2}{4T(M - 1)}\right]\right\} \tag{5}$$

where $E_1(X)$ is an exponential integral defined as

$$\left[E_1(X) = \int_X^\infty \frac{e^{-y}}{y} \, dy \right]$$

The Theis' well function for a non-leaky aquifer is equal to this exponential integral. The discrete kernel coefficient $\delta_r(M)$ is the response of a linear system at the end of the Mth unit time step to a unit pulse excitation given to the system during the first unit time step. In the coefficient $\delta_r(M)$, M is an index and it has no dimension, but the term 'M', which appears in the exponential integral $E_1[Sr^2/(4TM)]$, is an integer having the dimension of time. For computing the dimensionless term $Sr^2/(4TM)$, a transmissivity value corresponding to a unit time step size is used. In both the terms, $\delta_r(M)$ and $Sr^2/(4TM)$, values of M are numerically equal.

The large-diameter observation well acts as a recharge well in response to pumping in the production well. When several wells operate simultaneously, the resulting drawdown can be found by summing up the drawdowns caused by the pumping of individual wells because eqn. (1) is linear and the method of superposition is valid for a linear system.

Let the total time of pumping, t_p, be discretized to m units of equal time steps. The quantity of water, $Q_p(n)$, pumped during any time step n can be written as

$$Q_A(n) + Q_W(n) = Q_p(n) \tag{6}$$

in which $Q_A(n)$ is the water withdrawn from the aquifer storage through the production well during the nth time step, and $Q_W(n)$ is the water withdrawn from the production well storage during the nth time step.

For $n > m$, $Q_p(n) = 0$. Otherwise $Q_p(n)$ is equal to the rate of pumping during the nth time step.

The drawdown, $s_{wp}(n)$, in the water level at the production well at the end of the nth time step, due to abstraction from the production well storage, is given by

$$s_{wp}(n) = \frac{1}{\pi r_{cp}^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \tag{7}$$

where $Q_W(\gamma)$ represents rate of withdrawal from the production well storage or the replenishment during time step γ. $Q_W(\gamma)$ values are unknown a priori. A negative value of $Q_W(\gamma)$ means that there is a replenishment of the well storage which occurs during the recovery period.

Similarly, drawdown of the water surface at the observation well at the end of nth time step, due to recharge having taken place from the observation well storage to the aquifer, is given by

$$s_{\mathrm{wo}}(n) = \frac{1}{\pi r_{\mathrm{co}}^2} \sum_{\gamma=1}^{n} Q_{\mathrm{O}}(\gamma) \tag{8}$$

in which $Q_{\mathrm{O}}(\gamma)$ is the recharge rate from the observation well storage during time step γ.

The drawdown of the piezometric surface in the aquifer at the production well face at the end of the nth time step, due to abstraction from the aquifer through the production well and recharge from the observation well storage, is given by

$$s_{\mathrm{ap}}(n) = \sum_{\gamma=1}^{n} Q_{\mathrm{A}}(\gamma)\delta_{r_{\mathrm{wp}}}(n - \gamma + 1) - \sum_{\gamma=1}^{n} Q_{\mathrm{O}}(\gamma)\delta_{r_1}(n - \gamma + 1) \tag{9}$$

Drawdown of the piezometric surface in the aquifer at the observation well face is given by

$$s_{\mathrm{ao}}(n) = \sum_{\gamma=1}^{n} Q_{\mathrm{A}}(\gamma)\delta_{r_1}(n - \gamma + 1) - \sum_{\gamma=1}^{n} Q_{\mathrm{O}}(\gamma)\delta_{r_{\mathrm{wo}}}(n - \gamma + 1) \tag{10}$$

Because $s_{\mathrm{ap}}(n) = s_{\mathrm{wp}}(n)$, therefore

$$\sum_{\gamma=1}^{n} Q_{\mathrm{A}}(\gamma)\delta_{r_{\mathrm{wp}}}(n - \gamma + 1) - \sum_{\gamma=1}^{n} Q_{\mathrm{O}}(\gamma)\delta_{r_1}(n - \gamma + 1) = \frac{1}{\pi r_{\mathrm{cp}}^2} \sum_{\gamma=1}^{n} Q_{\mathrm{W}}(\gamma) \tag{11}$$

Rearranging

$$Q_{\mathrm{A}}(n)\delta_{r_{\mathrm{wp}}}(1) - \frac{1}{\pi r_{\mathrm{cp}}^2} Q_{\mathrm{W}}(n) - Q_{\mathrm{O}}(n)\delta_{r_1}(1) = \frac{1}{\pi r_{\mathrm{cp}}^2} \sum_{\gamma=1}^{n-1} Q_{\mathrm{W}}(\gamma)$$
$$+ \sum_{\gamma=1}^{n-1} Q_{\mathrm{O}}(\gamma)\delta_{r_1}(n - \gamma + 1) - \sum_{\gamma=1}^{n-1} Q_{\mathrm{A}}(\gamma)\delta_{r_{\mathrm{wp}}}(n - \gamma + 1) \tag{12}$$

In addition, $s_{\mathrm{ao}}(n) = s_{\mathrm{wo}}(n)$. Therefore

$$\sum_{\gamma=1}^{n} Q_{\mathrm{A}}(\gamma)\delta_{r_1}(n - \gamma + 1) - \sum_{\gamma=1}^{n} Q_{\mathrm{O}}(\gamma)\delta_{r_{\mathrm{wo}}}(n - \gamma + 1) = \frac{1}{\pi r_{\mathrm{co}}^2} \sum_{\gamma=1}^{n} Q_{\mathrm{O}}(\gamma) \tag{13}$$

Rearranging

$$Q_{\mathrm{A}}(n)\delta_{r_1}(1) - Q_{\mathrm{O}}(n)\left[\delta_{r_{\mathrm{wo}}}(1) + \frac{1}{\pi r_{\mathrm{co}}^2}\right] = \frac{1}{\pi r_{\mathrm{co}}^2} \sum_{\gamma=1}^{n-1} Q_{\mathrm{O}}(\gamma)$$
$$+ \sum_{\gamma=1}^{n-1} Q_{\mathrm{O}}(\gamma)\, \delta_{r_{\mathrm{wo}}}(n - \gamma + 1) - \sum_{\gamma=1}^{n-1} Q_{\mathrm{A}}(\gamma)\delta_{r_1}(n - \gamma + 1) \tag{14}$$

Equations (6), (12) and (14) can be expressed in the following matrix form:

$$\begin{bmatrix} 1 & 1 & 0 \\ \delta_{r_{wp}}(1) & -\dfrac{1}{\pi r_{cp}^2} & -\delta_{r_1}(1) \\ \delta_{r_1}(1) & 0 & -\left[\delta_{r_{wo}}(1) + \dfrac{1}{\pi r_{co}^2}\right] \end{bmatrix} \cdot \begin{bmatrix} Q_A(n) \\ Q_W(n) \\ Q_O(n) \end{bmatrix}$$

$$= \begin{bmatrix} Q_p(n) \\ \dfrac{1}{\pi r_{cp}^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) + \sum_{\gamma=1}^{n-1} Q_O(\gamma)\delta_{r_1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\delta_{r_{wp}}(n-\gamma+1) \\ \dfrac{1}{\pi r_{co}^2} \sum_{\gamma=1}^{n-1} Q_O(\gamma) + \sum_{\gamma=1}^{n-1} Q_O(\gamma)\delta_{r_{wo}}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\delta_{r_1}(n-\gamma+1) \end{bmatrix} \qquad (15)$$

In particular, for the first time step, the right-hand-side vector is $[Q_p(1), 0, 0]'$. $Q_A(n)$, $Q_W(n)$ and $Q_O(n)$ can be solved in succession starting from the first time step. Once $Q_A(n)$, $Q_W(n)$ and $Q_O(n)$ are known, the drawdown at any point in the aquifer at a distance 'r' from the production well can be found using the relation

$$s(r, n) = \sum_{\gamma=1}^{n} \delta_r(n-\gamma+1)Q_A(\gamma) - \sum_{\gamma=1}^{n} \delta_{r_2}(n-\gamma+1)Q_O(\gamma) \qquad (16)$$

in which r_2 is the distance between the point under consideration and the large-diameter observation well.

RESULTS AND DISCUSSION

The discrete kernel coefficients $\delta_{r_{wp}}(M)$, $\delta_{r_1}(M)$ and $\delta_{r_{wo}}(M)$ have been generated for known values of transmissivity, storage coefficient, radii of the production and the observation well screens, and the distance between the production well and the observation well. The exponential integrals have been evaluated using the polynomial and rational approximations given by Gautschi and Cahill (1964). The computational efficiency of these approximations has been highlighted by Huntoon (1980). Using a matrix inversion technique, eqn. (15) is solved for $Q_A(n)$, $Q_W(n)$ and $Q_O(n)$ in succession, starting from the first time step for known values of r_{cp}, r_{co} and $Q_p(n)$. The drawdowns of water level in the production well and observation well are obtained using eqns. (7) and (8), respectively, and have been computed for a constant pumping rate, Q_p.

The sensitivity analysis of drawdown to size and number of time steps has already been presented by Mishra and Chachadi (1985). In the present analysis, a time step size of $t/10$ has been adopted to compute the response at time, t.

The variations of $s_{wp}(t)/[Q_p/(4\pi T)]$ with $4Tt/(Sr_{wp}^2)$ at the production well and $s_{wo}(t)/[Q_p/(4\pi T)]$ with $4Tt/(Sr_1^2)$ at the observation well are shown in Figs. 2–5 for different values of α_p and α_o, where $\alpha_p = S(r_{wp}/r_{cp})^2$ and $\alpha_o = S(r_{wo}/r_{co})^2$. The parameters α_p and α_o quantify the storage of the production well and the observation well respectively. The results presented in Figs. 2 and 4 are for an observation well, which is located at a distance of 135 r_{wp} from the production well; those presented in Figs. 3 and 5 are for $r_1/r_{wp} = 100$. Variables $s_{wp}(t)$ and $s_{wo}(t)$ are the drawdowns at the production well and the observation well face, respectively, at time t; $s_{wp}(t)/[Q_p/(4\pi T)]$ and $s_{wo}(t)/[Q_p/(4\pi T)]$ can be regarded as the well functions for the large-diameter production well and observation well, respectively. The non-dimensional time–drawdown curves shown in Figs. 2–5 contain the response of the aquifer both during abstraction and the recovery phase. The recovery curve deviates from the time–drawdown curve of the abstraction phase at a particular non-dimensional time factor $4Tt_p/(Sr_{wp}^2)$, which is the non-dimensional duration of pumping. The non-dimensional time factor $4Tt_p/(Sr_{wp}^2)$ can be used to check the accuracy of the aquifer parameters determined by curve matching.

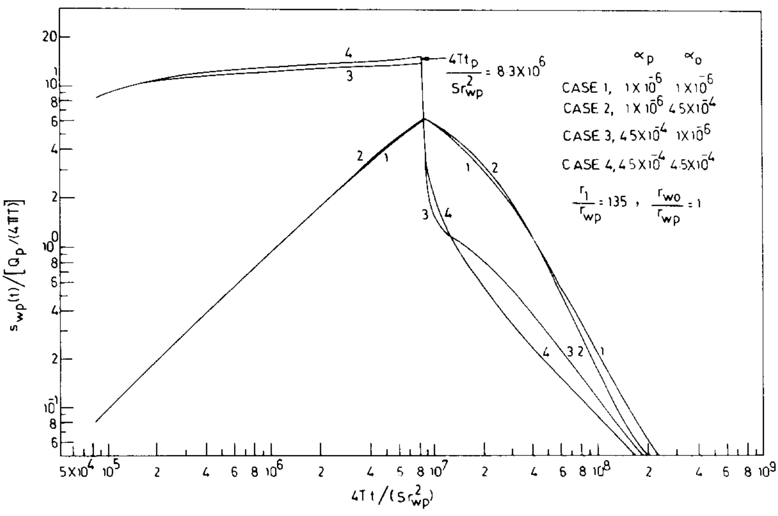

Fig. 2. Variation of $s_{wp}(t)/[Q_p/(4\pi T)]$ with $4Tt/(Sr_{wp}^2)$ at the production well for $r_1/r_{wp} = 135$ and $r_{wo}/r_{wp} = 1$.

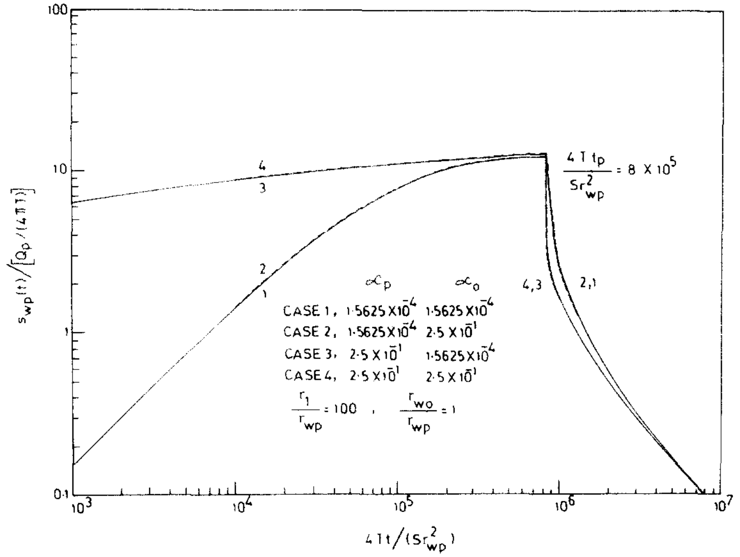

Fig. 3. Variation of $s_{wp}(t)/[Q_p/(4\pi T)]$ with $4Tt/(Sr^2_{wp})$ at the production well for $r_1/r_{wp} = 100$ and $r_{wo}/r_{wp} = 1$.

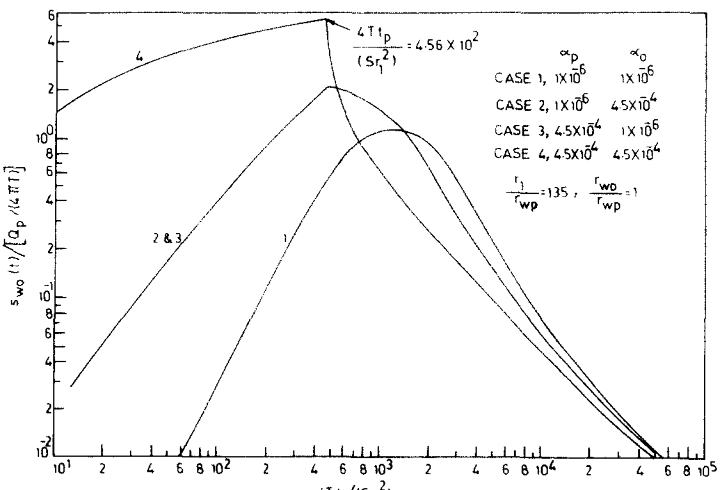

Fig. 4. Variation of $s_{wo}(t)/[Q_p/(4\pi T)]$ with $4Tt/(Sr_1^2)$ at the observation well for $r_1/r_{wp} = 135$ and $r_{wo}/r_{wp} = 1$.

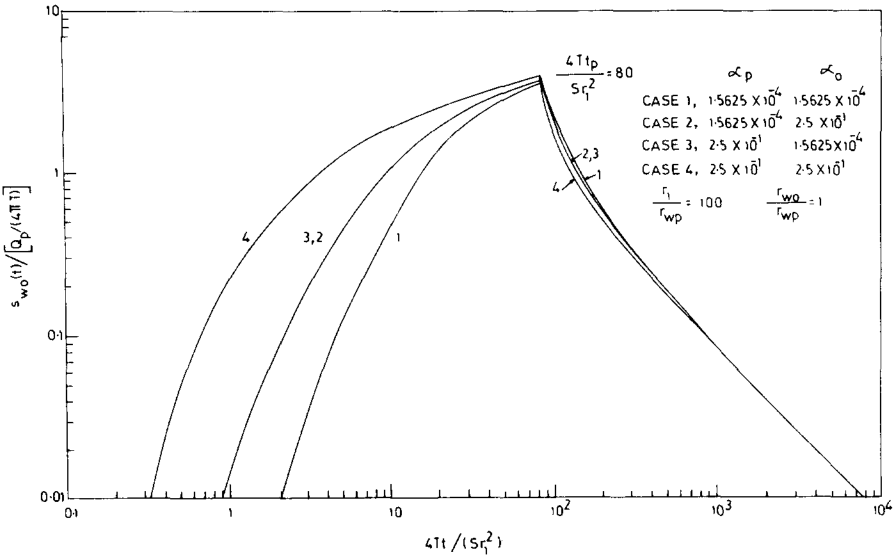

Fig. 5. Variation of $s_{wo}(t)/[Q_p/(4\pi T)]$ with $4Tt/(Sr_1^2)$ at the observation well for $r_1/r_{wp} = 100$ and $r_{wo}/r_{wp} = 1$.

The behaviour of the non-dimensional time–drawdown plots is discussed for four cases.

Case 1

In this case both the production well and the observation well are of large diameter having storage. The non-dimensional time–drawdown plots at the production well and the observation wells are shown in Figs. 2 and 4 (case 1), respectively, for α_p and $\alpha_o = 1 \times 10^{-6}$ and $4Tt_p/(Sr_{wp}^2) = 8.3 \times 10^6$.

In Fig. 2 the near straight-line portion of the time–drawdown graph during pumping is due to the influence of the production well storage. Because most of the pumped water comes from the production well storage, during the initial stage of pumping, the time–drawdown graph at the production well follows a straight line.

After the cessation of pumping, the production well soon starts recovering. On the other hand, the water level in the observation well continues to fall (Fig. 4). This is because when pumping is stopped there is a difference between the water level at the production well and the observation well. Because the water level in the observation well is higher, the flow of water from it towards the pumping well continues until the piezometric heads at the wells are equal. At this juncture the observation well also starts recovering its storage. During the initial period of recovery of the production well, water is withdrawn both from the observation well storage and aquifer storage. Therefore, the recovery rate of the production well during this period is higher than the recovery rate during the later period when both the wells start recovering. The water derived from the aquifer is distributed to replenish both wells during the later part of the recovery period.

Case 2

In this case only the production well is of large diameter having storage; the observation well is of small diameter with negligible storage.

The plots of non-dimensional time–drawdown curves at the production well and the observation well are shown in Figs. 2 and 4 (case 2) for α_p and α_o equal to 1×10^{-6} and 4.5×10^{-4}, respectively, and $4Tt_p/(Sr_{wp}^2) = 8.3 \times 10^6$. The near straight-line portion of the time–drawdown graph during the pumping phase at the production well is due to the influence of the well storage in the production well. During the early part of the pumping phase, most of the water pumped is derived from the production well storage, for which the time–drawdown graph at the production well is linear. If pumping continues for a long period, the well storage contribution reduces and aquifer contribution becomes dominant.

After the cessation of pumping, the production well soon starts to recover. On the other hand, the recovery in the observation well is delayed, or it occurs at a slower rate in comparison with that in the production well, because the gradient at the production well is steeper than that at the observation well.

Case 3

In this case the production well is of negligible diameter and the observation well is of large diameter having storage.

The plots of non-dimensional time–drawdown curves at the production well and at the observation well are shown in Figs. 2 and 4 (case 3) for values of α_p and α_o equal to 4.5×10^{-4} and 1×10^{-6}, respectively, and $4Tt_p/(Sr_{wp}^2) = 8.3 \times 10^6$. From Fig. 2 it is seen that the effect of the observation well storage on drawdown in the production well is to reduce the drawdown and the reduction only becomes prominent after pumping has occurred for some time.

After the cessation of pumping, the production well soon starts to recover. The rate of recovery in the production well is faster during the early period than the later period owing to a high gradient near the production well and the contribution of the observation well storage to the aquifer during the early part of recovery.

An important observation which is clear from Fig. 4 is that the time–drawdown responses at the observation well for cases 2 and 3, both during pumping and recovery, are identical. This indicates that when a production well of large diameter is pumped, the drawdown response in a well of negligible diameter is the same as that when the roles of the wells are reversed. This fact was first highlighted by Barker (1984).

Case 4

In this case both the production and the observation wells are of small diameter having negligible storage.

The values of α_p and α_o are equal to 4.5×10^{-4} and $4Tt_p/(Sr_{wp}^2) = 8.3 \times 10^6$. The effect of the well storage on drawdowns at both the wells during pumping and recovery being negligible, the time–drawdown graph follows the Theis' curve.

The non-dimensional time–drawdown plots for other values of α_p and α_o have been presented in Figs. 3 and 5 for all four cases. Smaller values of α_p and α_o mean higher well storage. From these plots it is seen that, as α_p and α_o values increase, drawdowns at the production well for cases 1 and 2 are nearly equal. Furthermore, for cases 3 and 4, drawdowns at the production well exhibit negligible difference.

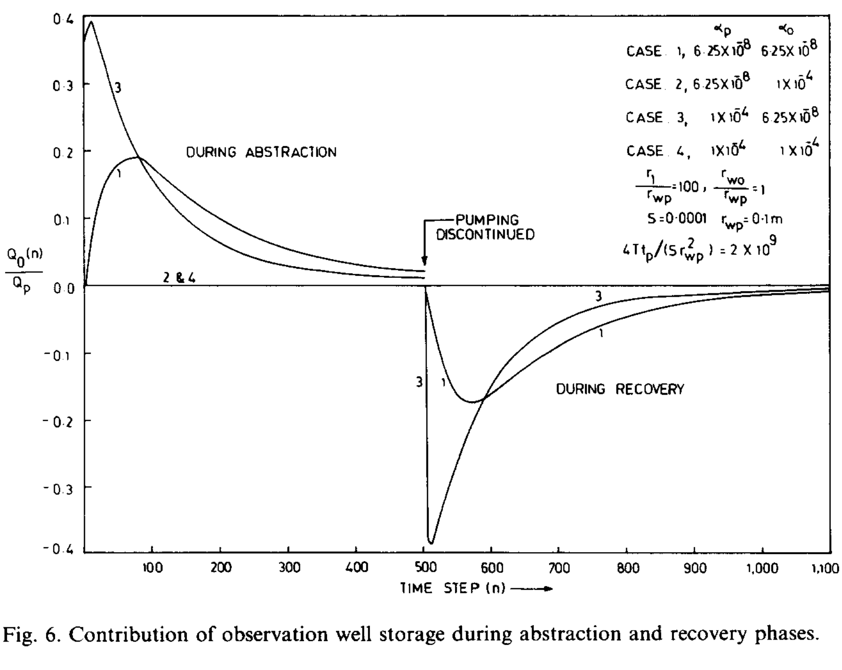

Fig. 6. Contribution of observation well storage during abstraction and recovery phases.

The contribution of the observation well storage during pumping, and the replenishment of the storage which occurs during recovery, are presented in Fig. 6 for all four cases. The results presented have been computed for $T = 500\,\mathrm{m^2\,day^{-1}}$, $S = 0.0001$, duration of pumping = 1 day, unit time step size = (1/500)th of a day, $r_1 = 10\,\mathrm{m}$, $r_{wp} = 0.1\,\mathrm{m}$, $r_{cp} = 0.1$ or $4.0\,\mathrm{m}$, $r_{wo} = 0.1\,\mathrm{m}$, $r_{co} = 0.1$ or $4.0\,\mathrm{m}$. The pumping was discontinued after 500 time steps. It can be seen from Fig. 6 that during the early period of pumping a larger quantity of water flows from the observation well into the aquifer in case 3 compared with case 1. At the beginning of recovery the replenishment of observation well storage is faster in case 3 than in case 1.

CONCLUSION

Unsteady flow, to a large-diameter production well and to a large-diameter observation well in a confined aquifer, was analysed by the discrete kernel approach. A numerical approximation has been derived to determine the contribution from the production well storage and the observation well storage, and to determine drawdown at any point in the aquifer. Non-dimensional time–drawdown curves comprising the response of the aquifer during the recovery phase are presented for specific cases. It was found that the influence of the observation well storage is more pronounced during recovery than during the abstraction phase. The effect of observation well storage increases with increasing observation well diameter. It was confirmed that the drawdown in an observation well of negligible diameter due to pumping in a large-diameter well is the same if the roles of the wells are reversed. The contribution from the observation well storage to the aquifer during abstraction is a function of the dimensions of the production well and the observation well, and of the time since pumping commenced. The contribution of the observation well storage increases from an initial zero value to a maximum value during pumping and then decreases as pumping is continued. Similar trends are observed during the recovery phase.

REFERENCES

Barker, J.A., 1984. Pumping tests using large-diameter observation wells. J. Hydrol., 67: 375–379.

Carslaw, H.S. and Jaeger, J.C., 1959. Conduction of Heat in Solids. Oxford University Press, New York, 510 pp.

Fenske, P.R., 1977. Radial flow with discharging well and observation well storage. J. Hydrol., 32: 87–96.

Gautschi, W. and Cahill, W.F., 1964. Exponential integral and related functions. In: M. Abramowitz and I.A. Stegun (Editors), Handbook of Mathematical Functions. Dover, New York, pp. 227–251.

Hantush, M.S., 1964. Hydraulics of wells. In: V.T. Chow (Editor), Advances in Hydroscience, Vol. 1. Academic Press, New York, pp. 331 and 339.

Huntoon, P.W., 1980. Computationally efficient polynomial approximations used to program the Theis equation. Ground Water, 18(2): 134–137.

Mishra, G.C. and Chachadi, A.G., 1985. Analysis of flow to large-diameter well during the recovery period. Ground Water, 23(5): 646–651.

Morel-Seytoux, H.J., 1975. Optimal legal conjunctive operation of surface and ground water. Second World Congress, International Water Resources Association, New Delhi, IV, pp. 119–129.

Mucha, I. and Paulikova, E., 1986. Pumping test using large-diameter production and observation wells. J. Hydrol., 89: 157–164.

Muskat, M., 1937. The Flow of Homogeneous Fluids through Porous Media. McGraw Hill, New York, 763 pp.

Papadopulos, I.S. and Cooper, H.H., 1967. Drawdown in a well of large diameter. Water Resour. Res., 3(1): 241–244.

Patel, S.C. and Mishra, G.C., 1983. Analysis of flow to large-diameter well by a discrete kernel approach. Ground Water, 21(5): 573–576.