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UNSTEADY FLOW TO A LARGE-DIAMETER WELL IN A FINITE AQUIFER

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Abstract

Using a discrete kernel approach unsteady flow to a large-diameter well in a finite confined aquifer during pumping and recovery has been analysed .It is found from the study that existence of a barrier boundary at a finite distance from the well has little influence on contribution of the well storage to pumping, but the barrier boundary affects drawdown significantly.

Introduction

Most of the solutions presented for the analysis of unsteady flow to a well in different hydrogeological conditions are based on the assumption that the aquifers are of infinite areal extent. Although aquifers of infinite areal extent do not exist, many aquifers are of such wide extent that, for all practical purposes they can be regarded as infinite.In case there exits an impervious barrier or a fully penetrating river near a well, the aquifer is to be treated as semi infinite.Analyses of unsteady flow to a well with negligible storage,located at the centre of a circular aquifer of finite areal extent have been made by Muskat (1937) and Kuiper (1972). Sen (1982) has presented a solution for unsteady flow to a large-diameter well having storage and located near a straight impervious boundary in an aquifer of semi infinite areal extent. The present study presents a solution for unsteady flow to a large-diameter well located at the centre of a finite aquifer that is limited by a circular barrier boundary.

Statement of the Problem

A schematic plan and a section of a large-diameter well in a homogeneous, isotropic, and confined aquifer of finite areal extent are shown in Fig.(1). It is assumed that the aquifer prior to pumping was at equilibrium condition. The well is located at the centre of the aquifer, which is limited by a circular barrier boundary at a distance 'a_1' from the centre of the well. The radius of the well screen is r_w, and that of the well casing is

r_c. Pumping is carried out at a uniform rate up to time t_p. The problem is to determine the drawdowns at the well face, at the barrier boundary, and at other points in the aquifer during pumping and recovery periods besides to find the aquifer's contribution and well storage's contribution during pumping and replenishment of the well storage during recovery.

Analysis

The assumptions made in the analysis are:(i) the time parameter is discrete;(ii) within each time step, the aquifer contribution and the well storage contribution are separate constant, but they vary from step to step; (iii) at any time the drawdown in the aquifer at the well face is equal to that in the well.

Boussinesq's partial differential equation, which describes the evolution of piezometric surface in a homogeneous, isotropic, and confined aquifer for an axially- symmetric radial flow onset by pumping of a well, is:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t}, \quad r > r_w \qquad \ldots(1)$$

in which r_w= radius of the well screen, s = drawdown in piezometric surface at a distance 'r' from the well at time 't', T = transmissivity, and ϕ = storage coefficient of the aquifer. The solution to the above differential equation has to satisfy the following boundary conditions in order to account for the well storage effect:

$$2\pi r_w T \frac{\partial s}{\partial r}\Big|_{r=r_w} - \pi r_c^2 \frac{\partial s_w}{\partial t} = -Q_p(t) \qquad \ldots(2)$$

in which $Q_p(t)$ = pumping rate at time 't'.Since it is assumed that the drawdown in the large-diameter well is equal to the drawdown in the aquifer at the well face ,the solution also needs to satisfy the following boundary condition at the well face :

$$s(r_w,t) = s_w(t) \qquad \ldots(3)$$

in which $s_w(t)$ = drawdown in the well.

The boundary condition to be satisfied to account for the existence of the no flow boundary at the radial distance 'a_1' is

$$\frac{\partial s}{\partial r}\Big|_{r=a_1} = 0 \qquad \ldots(4)$$

The initial condition required to be satisfied is

$$s(r,0) = 0 \qquad \ldots(5)$$

Discretising the time parameter by uniform time steps and assuming that the excitation to the system and its response are piece wise constants in each time-step, the alternate form of the boundary conditions stated in equation (2) is

$$Q_a(n) + Q_w(n) = Q_p(n) \qquad \ldots(6)$$

in which $Q_a(n)$ = aquifer contribution to pumping during n^{th} time-step, $Q_w(n)$ = well storage contribution to pumping during n^{th} time-step, and $Q_p(n)$ = pumping rate during n^{th} time-step. $Q_a(\gamma)$, and $Q_w(\gamma)$, for $\gamma=1,2,..n$, are unknown a priori.

Solution to the differential equation (1) for negligible well storage i.e. for $r_c= r_w$ and $r_w\rightarrow 0$, has been given by Muskat (1937) and Kuiper (1972) for continuous pumping of a well at a constant rate in a finite aquifer. Their solution is:

$$s(r,t) =-Q_p/(2\pi T)\Big[\frac{3}{4} + \log_e(r/a_1) - \frac{1}{2}\{(r/a_1)^2 + 4Tt/(\phi\, a_1^2)\}$$

$$+ 2\sum_{m=1}^{\infty}\Big\{(\alpha_m a_1) J_0(\alpha_m a_1)\Big\}^{-2} J_0(\alpha_m r)\ \exp\Big\{-\alpha_m^2 Tt/\phi\Big\}\Big] \qquad \ldots(7)$$

This solution satisfies the boundary condition stated in equation (4), and the initial condition stated in equation (5). If $r_c=r_w$, and $r_w\rightarrow 0$, the solution also satisfies the boundary condition stated in equation (2). $(\alpha_m a_1)$ values for m=1,2,3 ..., are the zeros of J_1, the Bessel's function of the first kind and of the first order. $(\alpha_m a_1)$ values have been tabulated for values of 'm' up to 20 (Abramowitz and Stegun, 1970). $(\alpha_m a_1)$ for higher values of 'm' can be evaluated using the following formula of McMahon's expansions for large zeros (Abramowitz and Stegun, 1970):

$$(\alpha_m a_1) \simeq \eta - \frac{\mu-1}{\eta} - \frac{4(\mu-1)(7\mu-31)}{3(8\eta)^3} - \frac{32(\mu-1)(83\mu^2-982\mu-3779)}{15(8\eta)^5}$$

$$- \frac{64(\mu-1)(6949\mu^3-153855\mu^2+158574\mu-6277237)}{105(8\eta)^7} \qquad \ldots(8)$$

in which $\eta = (m+\frac{1}{4})\pi$, and $\mu=4$.

Let $K(r,t)$ be the drawdown in the piezometric surface at a radial distance 'r' from the well in a confined aquifer of finite areal extent due to a unit step excitation. Substituting Q_p by 1, expression for $K(r,t)$ can be obtained from equation (7). Let $\delta_r(I)$ be the aquifer response at the end of I^{th} time-step to a unit pulse excitation given to the aquifer during the first time- step. The coefficient $\delta_r(I)$, which is known as the discrete kernel coefficient, is related to the unit step response function and is given by (Morel-Seytoux et al., 1975):

$$\delta_r(I) = K(r,I) - K(r,I-1) \qquad \ldots(9)$$

Substituting $K(r,I)$ and $K(r,I-1)$ by their respective expressions in equation (9) and simplifying, the following expression for discrete kernel coefficient for drawdown for a confined aquifer of finite areal extent is obtained :

$$\delta_r(I) = \frac{1}{\pi\phi a_1^2} - \frac{1}{\pi T}\sum_{m=1}^{\infty}\left\{(\alpha_m a_1)J_0(\alpha_m a_1)\right\}^{-2} J_0(\alpha_m r)\exp\left\{\frac{-\alpha_m^2 TI}{\phi}\right\}$$
$$+ \frac{1}{\pi T}\sum_{m=1}^{\infty}\left\{(\alpha_m a_1)J_0(\alpha_m a_1)\right\}^{-2} J_0(\alpha_m r)\exp\left\{\frac{-\alpha_m^2 T(I-1)}{\phi}\right\}, \quad I>1 \qquad \ldots(10)$$

For $I=1$, $\delta_r(1)$ is given by

$$\delta_r(1) = -\frac{1}{2\pi T}\left[\frac{3}{4} + \log_e(r/a_1) - \frac{1}{2}\left\{(r/a_1)^2 + \frac{4T}{\phi a_1^2}\right\}\right.$$
$$\left. + 2\sum_{m=1}^{\infty}\left\{\alpha_m a_1 J_0(\alpha_m a_1)\right\}^{-2} J_0(\alpha_m r)\,\exp\left\{-(\alpha_m)^2\frac{T}{\phi}\right\}\right] \qquad \ldots(11)$$

Having obtained the discrete kernel coefficients for drawdown in a finite aquifer in response to pumping of a well which has negligible storage, the solution for a large-diameter well problem in a finite aquifer can be obtained as follows :

The drawdown in the piezometric surface at the well face in a confined aquifer at the end of n^{th} unit time step due to a varying withdrawal from aquifer storage is given by (Morel-Seytoux et al., 1975):

$$s(r_w,n) = \sum_{\gamma=1}^{n} Q_a(\gamma)\delta_{rw}(n-\gamma+1) \qquad \ldots(12)$$

Substituting 'r' by r_w, the coefficients $\delta_{rw}(I)$ can be obtained from equations (10)and (11). The drawdown in the water level in the well at the end of n^{th} unit time step due to withdrawal from well storage up to n^{th} time-step is given by:

$$s_w(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_w(\gamma) \qquad \ldots(13)$$

Since $s(r_w,n) = s_w(n)$, equations (12)and (13)are equal.Hence,

$$\frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_w(\gamma) = \sum_{\gamma=1}^{n} Q_a(\gamma)\delta_{rw}(n-\gamma+1) \qquad \ldots(14)$$

Splitting each of the summations into two parts,one part containing the summation up to $(n-1)^{th}$time-step and the other part containing the n^{th} term, and collecting the unknowns for the n^{th} time step, the following expression is obtained:

$$Q_a(n)\delta_{rw}(1) - \frac{1}{\pi r_c^2} Q_w(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) - \sum_{\gamma=1}^{n-1} Q_a(\gamma)\delta_{rw}(n-\gamma+1) \qquad \ldots(15)$$

$Q_w(n)$ and $Q_a(n)$ are solved from equation (6) and (15) and they are given by the following expressions :

$$Q_a(n) = \frac{Q_p(n) + \sum_{\gamma=1}^{n-1} Q_w(\gamma) - \pi r_c^2 \sum_{\gamma=1}^{n-1} Q_a(\gamma)\delta_{rw}(n-\gamma+1)}{1 + \pi r_c^2 \delta_{rw}(1)} \qquad \ldots(16)$$

$$Q_w(n) = \frac{\pi r_c^2 \delta_{rw}(1) Q_p(n) - \sum_{\gamma=1}^{n-1} Q_w(\gamma) + \pi r_c^2 \sum_{\gamma=1}^{n-1} Q_a(\gamma)\delta_{rw}(n-\gamma+1)}{1 + \pi r_c^2 \delta_{rw}(1)} \qquad \ldots(17)$$

In particular for time step 1, $Q_a(1) = Q_p(1)/[1 + \pi r_c^2 \delta_{rw}(1)]$, and $Q_w(1) = \pi r_c^2 \delta_{rw}(1) Q_p(1)/[1 + \pi r_c^2 \delta_{rw}(1)]$.

Using equations (16) and (17),$Q_a(n)$ and $Q_w(n)$ can be found in succession starting from time step 1. If the pumping period is discretised to 'm'units of equal time steps, $Q_p(n)=Q_p$ for $n \leq$ 'm', and $Q_p(n) = 0$ for $n >$ 'm'.From equations(16) and (17) the response of the aquifer and well storage replenishment can also be known for the recovery periods.

Results and Discussion

The discrete kernel coefficients, $\delta_r(I)$, are generated making use of equations(10) and (11) for assumed values of aquifer parameters T and ϕ and radii r_w and a_1. The converging series that appears in equation(10) has been truncated after the 100^{th} term. The large zeros, after the 10^{th} one, of the Bessel function have been evaluated using McMahon's formula given by equation(8). Values of small zeros have been taken from the tabulated values (Abramowitz & Stegun, 1970). For given radius of well casing, r_c, pumping rate, Q_p, and duration of pumping, t_p, $Q_a(n)$ and $Q_w(n)$ are found in succession starting from the first time-step.

The contributions of well storage and a finite aquifer, and dimensionless drawdowns at several observation wells, due to pumping of a well at a constant rate for a finite time period, are presented in Table 1 for a_1 equal to 500m. The flow characteristics, $Q_a(t)/Q_p$, $Q_w(t)/Q_p$, and $s_w(t)/[Q_p/(4\pi T)]$, for $a_1/r_w=\infty$, have been computed using discrete kernel approach (Mishra and Chachadi, 1985) and these are presented in Table 2. The data of Tables 1 and 2 establish that for a constant value of α, whether an impervious boundary exits at $a_1/r_w=5000$ or at $a_1/r_w=\infty$, the contributions of well storage towards pumping remain almost the same. Also up to nondimensional time 8×10^5, influence of the barrier boundary on the drawdown at the well is insignificant. The drawdown will be affected if the pumping is continued for a longer duration.

Variations of $Q_w(n)/Q_p$ and $Q_a(n)/Q_p$ with nondimensional time factor, $4Tt/(\phi\, r_w^2)$, during pumping and recovery periods are shown in Figure 2 for α =0.000625, and a_1/r_w= 10000 .The well storage parameter α is defined as $\alpha = \phi(r_w/r_c)^2$. It is found that both for a_1/r_w =5000 and 10000, at about $4Tt/(\phi\, r_w^2) = 9.5 \times 10^3$, the well storage contribution and the aquifer contribution are equal. This indicates that the barrier boundary has no influence on the performance of well storage.

Variations of nondimensional drawdown, $s(r,t)/[Q_p/(4\pi T)]$, with dimensionless time, $4Tt/(\phi\, r_w^2)$, at the pumping well, and near the barrier boundary are shown in Fig.(3) both for pumping and recovery phases for different durations of pumping. The variations show that immediately after cessation of pumping the drawdown at the pumped well decreases with time but the drawdown at the barrier boundary continues to increase even after the stoppage of pumping .This is because of the continuation of the aquifer's contribution to well storage. There is a permanent drawdown for each pumping operation because of the finite extent of the aquifer. Some time after the cessation of pumping the drawdowns at the pumped well and at the barrier boundary become equal indicating that the aquifer has come back to a rest condition after the stoppage of pumping.

Conclusions

Based on the study the following conclusions are drawn :

(i) The influence of a barrier boundary on the contribution of well storage towards pumping is negligible.

(ii) The drawdown at any location is influenced by the barrier boundary. For every pumping operation, there is a permanent drawdown because of the finite areal extent of the aquifer. The slope of the time-drawdown graph at the end of recovery is zero for any observation well in a finite aquifer.

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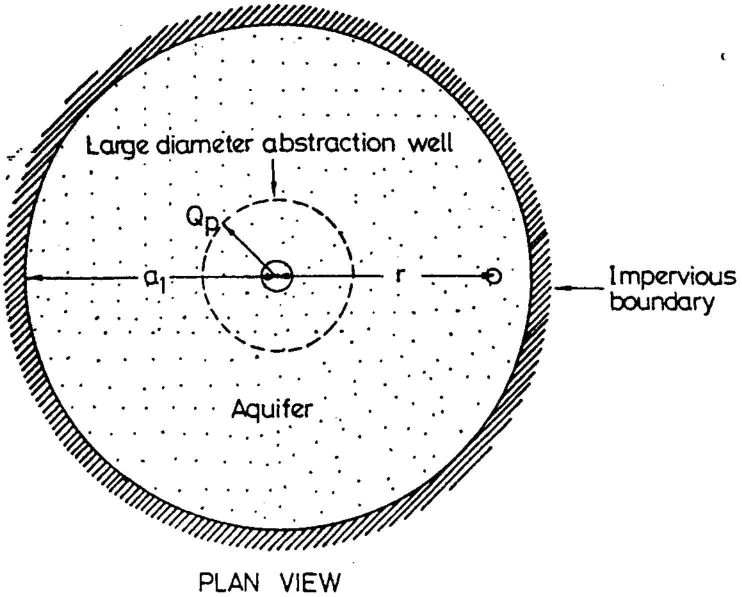
Table 1 Contributions of aquifer and well storage, and drawdowns evaluated for α =0.000025, a_1/r_w=5000, and $4Tt_p/(\phi r_w^2)=8\times10^5$ [r_c=2m, r_w=0.1m, a_1=500m, T=100m^2/day ϕ =0.01 and t_p=2 days]

Nondimensional time factor $4Tt/(\phi r_w^2)$	Q_a/Q_p	Q_w/Q_p	Nondimensional Drawdowns, $s(r,t)/[Q_p/(4\pi T)]$, at r/r_w= 1	2500	5000
2 x 10^4	0.05535	0.94465	0.4827	0.0000020	0.0000004
4	0.10123	0.89877	0.9411	0.0000070	0.0000008
6	0.14261	0.85739	1.3779	0.0001000	0.0000011
8	0.18073	0.81927	1.7951	0.0005060	0.0000014
10	0.21620	0.78380	2.1939	0.0014890	0.0000017
30	0.47735	0.52265	5.3886	0.0629840	0.0000039
50	0.63722	0.36278	7.5642	0.1975100	0.0000052
80	0.77911	0.22089	9.6921	0.4473700	0.0000063
Pumping Stopped					
82	0.73053	-0.73053	9.3178	0.4644300	0.0000060
84	0.69116	-0.69116	8.9645	0.4814700	0.0000057
86	0.65605	-0.65605	8.6296	0.4983700	0.0000054
88	0.62397	-0.62397	8.3113	0.5149200	0.0000052
90	0.59431	-0.59431	8.0083	0.5308400	0.0000049
100	0.47232	-0.47232	6.6891	0.5956600	0.0000039
120 x10^4	0.31003	-0.31003	4.7759	0.6516600	0.0000025

Table 2 Contributions of aquifer and well storage, and drawdowns evaluated for α =0.000025, $a_1/r_w=\infty$, and $4Tt_p/(\phi r_w^2)=8\times10^5$ [r_c=2m, r_w=0.1m, T=100m^2/day ϕ =0.01 and t_p=2 days]

Nondimensional timefactor, $4Tt/(\phi r_w^2)$	Q_a/Q_p	Q_w/Q_p	Nondimensional Drawdown $s_w(t)/[Q_p/(4\pi T)]$
2 x 10^4	0.0553	0.9446	0.4827
4	0.1012	0.8988	0.9410
6	0.1426	0.8574	1.3779
8	0.1807	0.8193	1.7950
10	0.2162	0.7838	2.1939
30	0.4774	0.5226	5.3886
50	0.6372	0.3628	7.5642
80 x10^4	0.7791	0.2209	9.6921



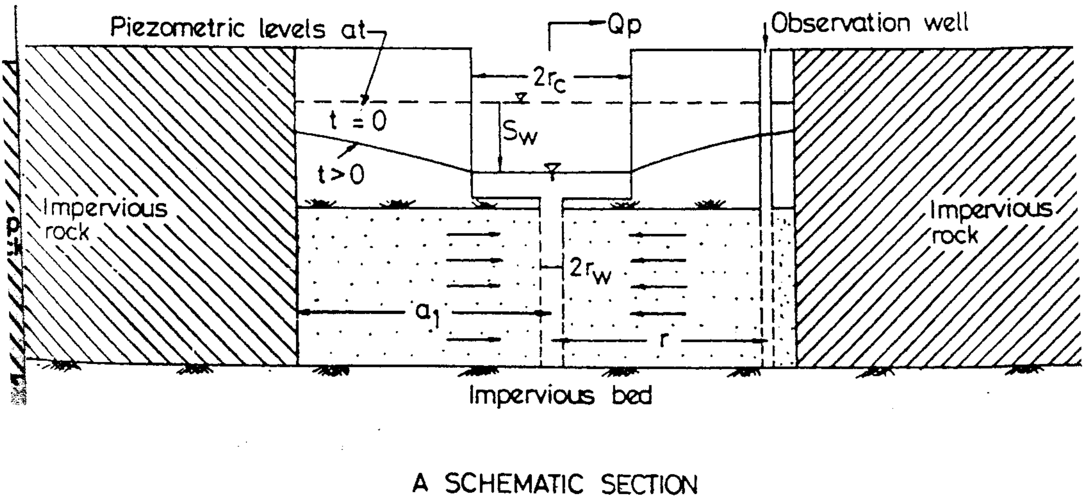


Fig.1 -Plan and Schematic Section of a Large-Diameter Well in a Finite Aquifer

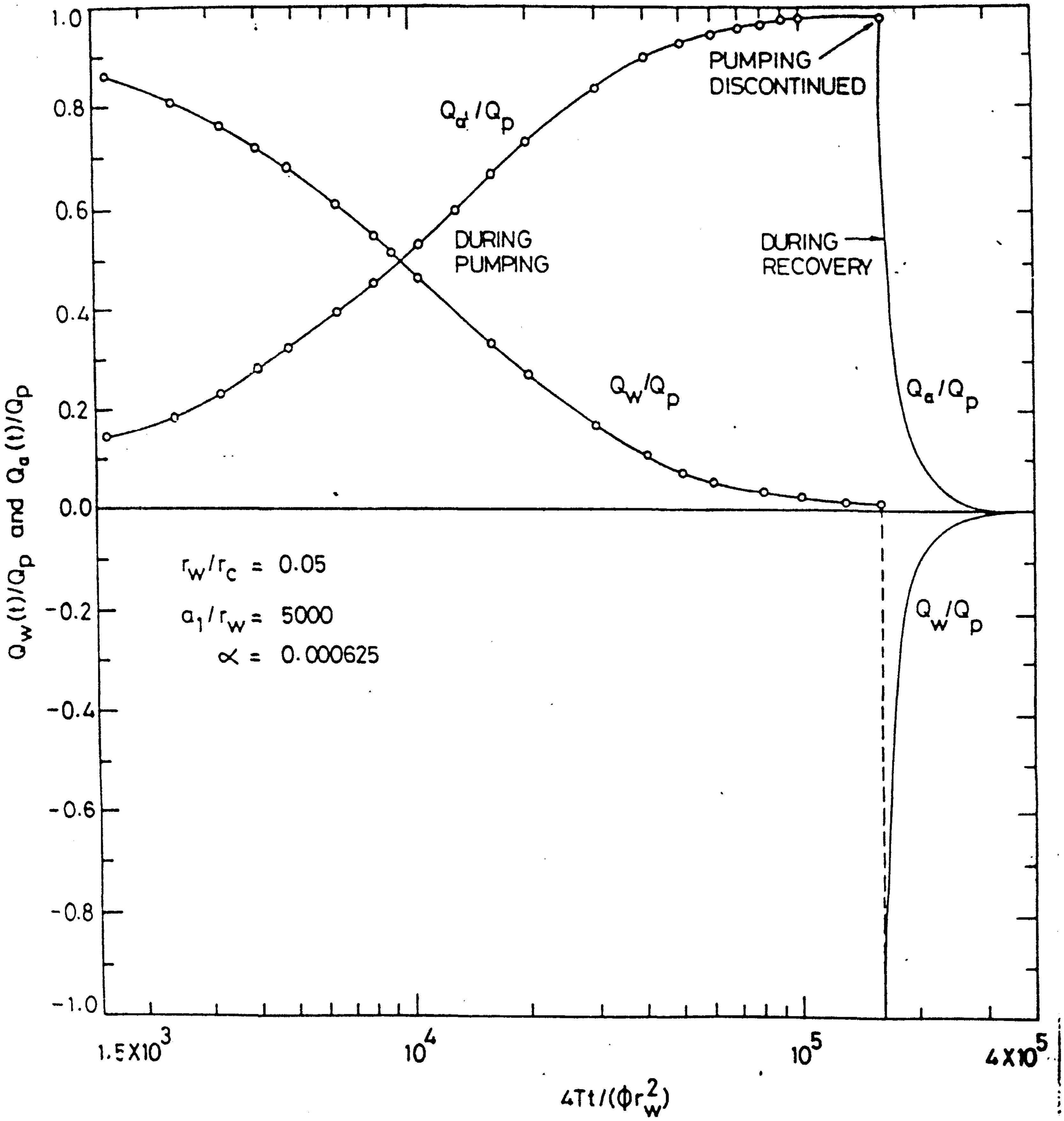


Fig.2- Variation of $Q_w(t)/Q_p$ and $Q_a(t)/Q_p$ with $4Tt/(\phi\ r_w^2)$ for $a_1/\ r_w$ =10000.

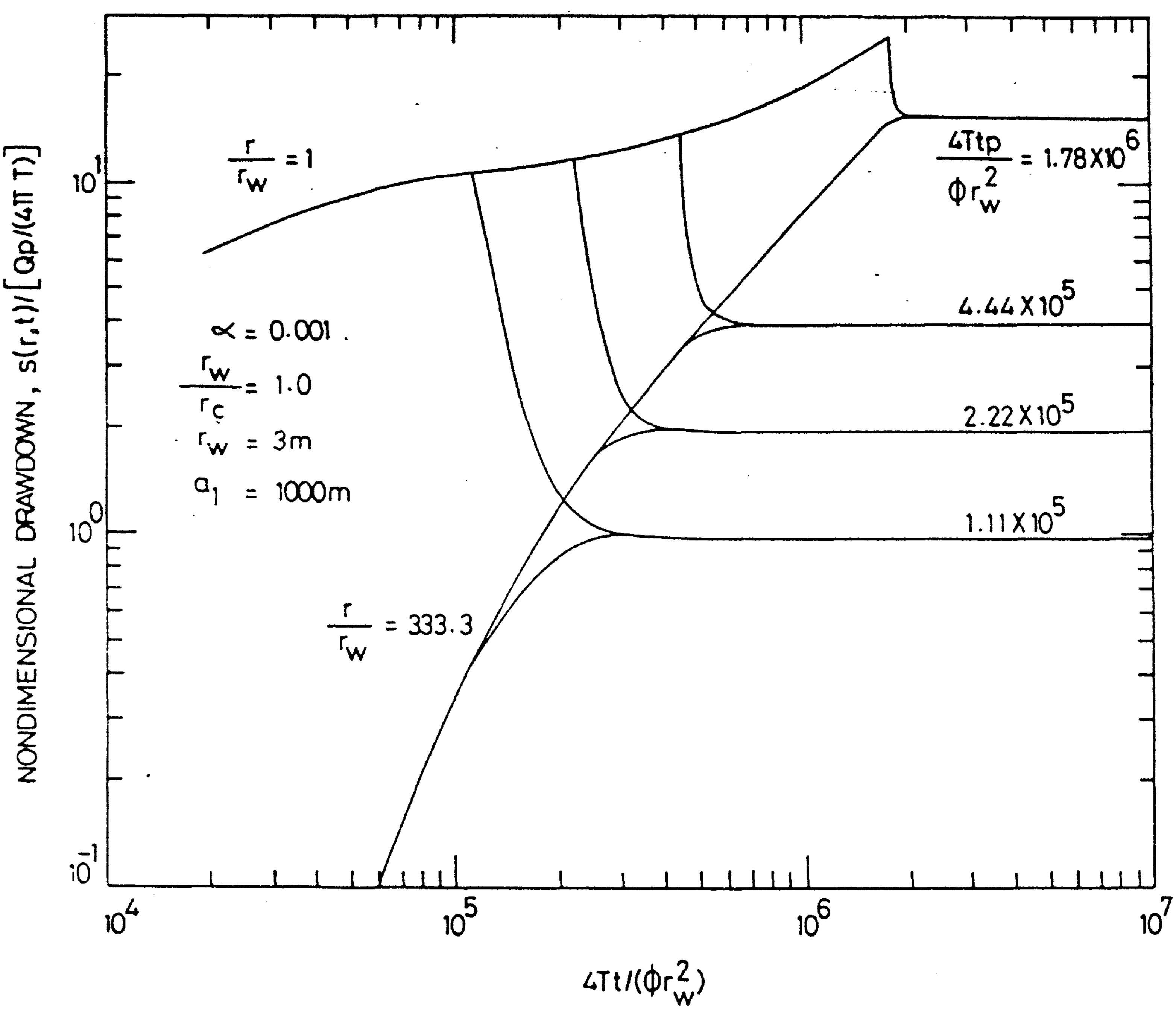


Fig.3 - Variation of nondimensional drawdown, $s(r,t)/[Q_p/(4\pi T)]$ with $4Tt/(\phi r_w^2)$ at $r/r_w=1$. and 333.33 evaluated for $\alpha=0.001$.and $a_1/r_w= 333.33$