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Differential Inequalities For A First Order Neutral Differential Equations

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Abstract

In this paper we present a comparison result to discuss the periodic boundary value problem for neutral differential equation with piecewise constant delay.

Keywords: Neutral Differential Equations, Piecewise constant argument, positive solution.

1. INTRODUCTION

In past few decades Neutral Differential Equations appeared in numerous models of Biology, Physics, Engineering, Control Systems, Mechanics, etc. These models turned out to be very useful in the situation where the system depends not only on the present state but also on the past states. In most of these applications delay function is either discrete or continuous. But there are some systems where the delay is piecewise continuous as reported for the first time in [1], [2] and further developed in [3], [4], [5].

This paper is concerned with the periodic boundary value problem (PBVP) of the following type is:

$$x'(t) = -M_1 x(t) - M_2 x([t]) - M_3 x'([t]), t \in J \quad (1.1)$$

$$x(0) = x(T), \quad (1.2)$$

where $J = [0,T], x \in C'(J,R), M_1 > 0$ and $M_2, M_3 \geq 0$ and [.]is the greatest integer function.

Let D denote the class of all functions $x: J \cup \{-1\} \to R,$ where $J = [0,T]$, satisfying

1. $x(-1) = x(0)$;
2. $x(t)$ is continuous, for $t \in J$;
3. $x'(t)$ exists and is continuous on the intervals $[n, n+1)$, for $n = 0,1,2,\dots,\hat{T}-2$ and on $[\hat{T}-1, T)$,

where
$\hat{T} = [T]+1, T \neq [T], and\ \hat{T} = T, T = [T]$.
A function $x: J \cup \{-1\} \to R$, is said to be solution of (1. 1) and (1. 2), if $x \in D$ and satisfies (1. 1) and (1. 2) with $x'(t) = x'_{+}(t) on\ t = 1,2,\dots \hat{T}-1$.

2. MAIN RESULT

In this section we present a comparison result for (1. 1) and (1. 2). Comparison resultsare useful in studying differential equations and its various properties.
Let $n \in R^{+}$, such that $T \in (n, n+1]$.Consider the space D on functions $w: J \to R$ such that $w \in C(J). w'(t)$ exists and is continuous on $[n-1, n), n = 0,1,2,\dots,\hat{T}-1$ and on $[\hat{T}-1, T]$.

Theorem 2. 1. Suppose that $w \in D$ such that:

- (Y1) $w'(t) + M_1 w(t) + M_2 w([t]) - M_3 w'([t]) \leq 0, t \in J$,
- (Y2) $w(0) \leq w(T)$,
- (Y3) $M_2 \geq M_1 M_3$,
- (Y4) $\left[\frac{M_2 - M_1 M_3}{1+M_3}\right]\left[\frac{e^{M_1}-1}{M_1}\right] < 1$,

where $M_1 > 0$, and $M_2, M_3 \geq 0$ are constants. Then $w(t) \leq 0, \forall t \in J$.

Proof: Let $q(t) = [w(t) + M_3 w([t])]e^{M_1 t}, t \in J$. Then, by (Y1), for every $t \in J$ we get

$$q'(t) = M_1 e^{M_1 t}[w(t) + M_3 w([t])] + e^{M_1 t}[w(t) + M_3 w([t])]' \tag{2.1}$$

$$\leq -\left[\frac{M_2 - M_1 M_3}{1+M_3}\right] e^{M_1 (t-[t])} q([t]). \tag{2. 2}$$

Consider $t \in [n-1, n), for\ n = 1,2,\dots,\hat{T}-1$, we can write

$$q(t) \leq q(n-1)\left\{1 - \left[\frac{M_2 - M_1 M_3}{1+M_3}\right]\left[\frac{e^{M_1 (t-n+1)} - 1}{M_1}\right]\right\}. \tag{2. 3}$$

For $t = n$ where $n = 1,2,\dots,\hat{T}-1$, we get

$$q(n) \leq q(n-1)\left\{1 - \left[\frac{M_2 - M_1 M_3}{1+M_3}\right]\left[\frac{e^{M_1} - 1}{M_1}\right]\right\}$$

i. e.

$$q(n) \leq N q(n-1) \tag{2. 4}$$

Where
$N = \left\{1 - \left[\frac{M_2 - M_1 M_3}{1 + M_3}\right]\left[\frac{e^{M_1} - 1}{M_1}\right]\right\}$
Consider $t \in [\hat{T} - 1, T]$ then we have,
$$q(t) \leq q(\hat{T} - 1)\left\{1 - \left[\frac{M_2 - M_1 M_3}{1 + M_3}\right]\left[\frac{e^{M_1(t - \hat{T} + 1)} - 1}{M_1}\right]\right\} \quad (2.\ 5)$$

For $t = T$
$$q(T) \leq q(\hat{T} - 1)\left\{1 - \left[\frac{M_2 - M_1 M_3}{1 + M_3}\right]\left[\frac{e^{M_1(T - \hat{T} + 1)} - 1}{M_1}\right]\right\} \quad (2.\ 6)$$
Now we are going to prove that it is not possible to have for every $n = 1,2, \dots, \hat{T} - 1$, $q(n) > 0$. (2. 7)
Let us assume relation (2. 7) holds, then from relation (2. 2) we can say q (t) is decreasing function on $[n - 1, n), n = 1,2, \dots, \hat{T} - 1$and on$[\hat{T} - 1, T]$.
Thus we have $q(t) \leq q(\hat{T} - 1) \dots \leq q(1) \leq q(0)$, and which gives $w(t) \leq e^{M_1 T} w(T) \leq w(0)$ which contradicts condition (Y2) of the theorem.
Therefore, we have $q(k) \leq 0$ for some $k = \{0,1,2, \dots, \hat{T} - 1\}$.From continuous application of relation (2. 4) we get $q(k + l) \leq N^l q(k) \leq q(k) \leq 0$where $l = 1,2, \dots, \hat{T} - 1$. In particular when $l = \hat{T} - k - 1$ we get $q(\hat{T} - 1) \leq 0$.From relation (2. 6) we get $q(T) \leq 0$ and using (Y2) we get $q(0) \leq 0$.
Again from relation (2. 4) we get $q(1) \leq 0$ and $q(n) \leq 0, n = 1,2, \dots, \hat{T} - 1$.From relation (2. 6) we also get $q(T) \leq 0$. Hence we get $q(t) \leq 0$for $t \in [n - 1, n)$ where $n = 1,2, \dots, \hat{T} - 1$ and $[\hat{T} - 1, T]$.
Hence by relations (2. 4) and (2. 6) we get q(t) decreasing on every $t \in [n - 1, n)$, $n = 1,2, \dots, \hat{T} - 1$and on $[\hat{T} - 1, T]$. Hence $q(t) \leq 0$ for every $t \in [0, T]$. Consequently we get $w(t) \leq 0, t \in J$.
Hence the proof.
We apply this result to establish the solution of (1. 1), (1. 2). If $x \in D$is a solution of PBVP (1. 1), (1. 2) then both $x(t)$ and $-x(t)$ satisfies the inequality (Y1) and (Y2). Hence we have the following corollaries.

Corollary 2. 2. The PBVP (1. 1) and (1. 2) has the unique solution$x(t) = 0$, if (Y3) and (Y4) holds.

Corollary 2. 3. Let $x \in D$ satisfying (Y1)and $x \leq 0$. If (Y3) and (Y4) are satisfied, then$x(t) \leq 0, \forall t \in J$.

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