Control Limit Models for Cash Management in Bank Branches

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Development of appropriate policies for maintaining adequate cash balance at the branches and transferring excess cash to the central office or to the needy branches are the major tasks undertaken by the commercial banks in India. At the branch level the problem is to decide the amount of cash to be drawn from or remitted to the link branch and the timings of the withdrawals or remittances. The link branches also face similar problems *vis-a-vis* central office. At the central office the problem is of deciding the amount to be invested in or withdrawn from short term assets and the timings of such investments and withdrawals.

The objective of the study is to identify appropriate models for determining decentralized control limit for cash balances at the branch and which minimizes holding cost of cash and transfer cost of cash. Two years daily cash flow data, from eight branches of a private sector scheduled commercial bank were used to verify the vaious hypotheses. Models were developed from the first year's data and were then applied to the second year's data for comparison.

In actual practice decisions regarding the amount and timings for withdrawal or remittances of cash are taken on subjective basis by the branch managers using the mechanism of 'retention limits' (i.e. upper limits) which are determined by the regional managers in consultation with the branch managers. The Miller and Orr model which was adopted by some of the banks, fared better than the actual practice, but fared significantly worse than the best policies that could be obtained post facto. The reason for this could be the drift in daily cash flows which is a violation of one of the basic assumptions of the model. The Baumol model which was adopted by some other banks to take care of the drift in daily cash flows gave good results, but fared worse than the method of enumeration. A method of enumeration for deriving bench mark policy for determining the limits of cash holdings based on the first year's data gave good results on the second year's data in comparison to the best policies that could be obtained post facto. Cash flow distribution was stable across the years. This method of enumeration being computationally simple, has greater potential for application in practical decision making.

Warrants : Some Issues

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Warrants are yet another type of financial instrument which touched Indian capital market of late. They are very important financial securities. A significant proportion of private placement bonds and a smaller portion of public issues are sold with warrants. Sometimes, warrants are attached to issues of preferred stocks or given to investment bankers as compensation for underwriting services. Occasionally bonds and warrants can be traded only as a package, but generally warrants are detachable. The warrant holder is not entitled to vote or to receive dividends, but the exercise price of warrant is adjusted for any stock dividend or stock splits. Warrant is a call option to buy a stated number of stock at specified price. There are many varieties of warrants.

The value of a warrant depends on : *(a)* the outlook for the stock price, *(b)* leverage effect, and *(c)* time remaining until expiry period. A warrant with no theoretical value can command a price because it provides a long term option on the stock. Prices of warrants are subject to

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Introduction

Development of policies for maintaining adequate cash balances in the branches and transferring of excess cash from surplus branches to the Central Office or needy branches are major tasks faced by the commercial banks in India. Some of the commercial banks adopt a hierarchical structure of three levels consisting of the Central Office, the link branches and other branches for operating the cash management policies. At the branch levels, the problem is to decide the amount of cash to be drawn from/remitted to the link branch and its timings. The link branches face a similar problem *vis-a-vis* the Central Office. At the Central Office, the problem is to decide the amount to be invested in/withdrawn from short term assets and their timings.

The objective of this study was to identify appropriate models for developing decentralized control limits for branch level cash management which minimize the relevant costs, *viz.* holding cost of cash and cash transfer cost.

Practice

Decisions regarding the amount and timings for remittances of excess cash are taken by branches using the mechanism of 'retention limits'. 'Retention limits' are the upper limits for cash holding by branches and

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they vary from branch to branch. Cash accruing above this limit, is to be remitted to the link branch. The limits are usually fixed by the regional managers, in consultation with the branch managers. The decisions regarding the amount and timings of withdrawals of cash are left to the branch managers themselves.

One of the largest public sector banks in India had recommended the Miller and Orr model[1] and the Baumol model[2] for determining the upper limits (retention limits) for cash holding in its branches. The upper limits in both these models were determined on the basis of a trade off between the holding cost of cash and cash transfer cost. One private sector scheduled commercial bank in Kerala is also reported to have used the Miller and Orr model for its branches.[3] However, there is no evidence as to whether the assumptions behind the models held good for the cash flow pattern of the banks concerned.

The models when formulated by the respective authors were meant for prediction of transactions demand for cash. Subsequently, they were also prescribed for control of cash in corporate situations. Corporate cash management situation is generically similar to cash management situation in banks especially at the branch level when we consider a branch as a profit centre. This has aroused interest on the part of banking industry as well as academic circles, in seriously considering the applicability of these models to bank branch situations.

In the present study, the Miller and Orr model as well as the Baumol model are tested using two years' daily cash flow data from seven branches of a private sector scheduled commercial bank. The models were developed from the first year's data and were then applied on the second year's data for comparisons. The seven branches chosen were in the same district and were all controlled by the same link branch.

The Models

Miller and Orr developed one of the earliest control limit models for the stochastic cash balance problem. The policy was assumed to be of the target threshold form. Whenever cash balance reached an upper threshold 'h' or a lower threshold 'u', it was brought to an intermediate target 'z' through cash transfers to/from a short term asset (Figure 1). With the objective of minimizing 'steady state' costs, expressions were derived for 'z' and 'h' under the following assumptions : *(a)* cash flows behaved as if generated by a Gaussian function with zero mean; *(b)* lumpy transfer costs, which did not vary with the amount transferred, were equal in both directions; and *(c)* holding costs/opportunity costs for cash balances were linear.

The solution was as follows :

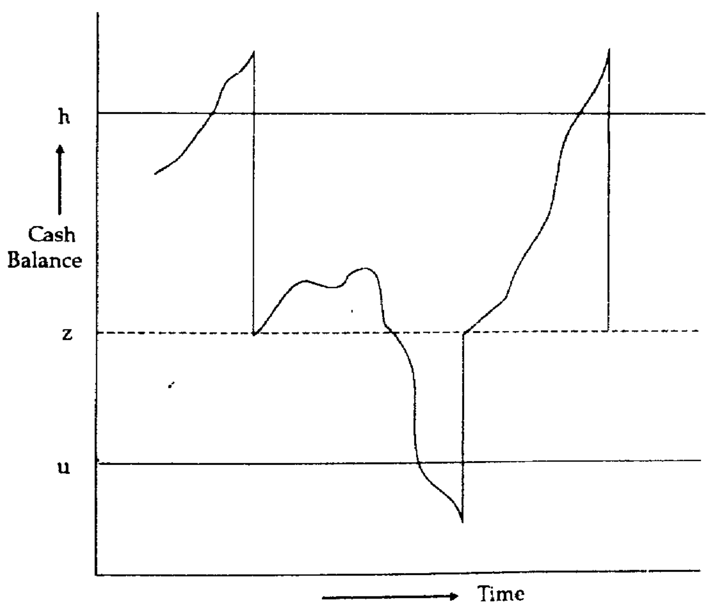
$$z^* = \left[\frac{3b\,\sigma^2}{4i}\right]^{1/3}$$

$$h^* = 3\left[\frac{3b\,\sigma^2}{4i}\right]^{1/3} + u$$

$$= 3z^* + u$$

where 'b' is the fixed cost of cash transfer in either direction, σ^2 is the variance of daily net cash flows and 'i' is the opportunity cost per day of 1 unit of idle cash.

Figure 1
Control Limit Policy



Baumol developed one of the earliest models of the transactions demand for cash. The model was developed under the assumptions of : *(a)* a known, continuous, uniform cash flow rate; *(b)* instantaneous replenishment of cash; *(c)* lumpy cost of replenishment; and *(d)* linear opportunity cost for holding cash.

The policy arrived at was equivalent to the simple Economic Order Quantity Policy of inventory control. The model was recommended by one of the largest public sector banks for situations of drift in cash flows.

The upper control limit obtained was :

$$h^* = \sqrt{\frac{2bt}{i}} + u$$

where 't' is the cash flow rate per day and 'b' and 'i' are as defined under the Miller and Orr model given above.

In both Miller and Orr and Baumol models, 'u' or the lower threshold was considered as given/determined outside the model. In the present study, the lower threshold for each branch was ascertained from the branch managers and was based on their normal practice (i.e. subjectively determined).

Validity of the Assumptions

The following assumptions had to be validated for using the Miller and Orr model for arriving at cash management policies : *(a)* independence of cash flows across days; *(b)* stability of the cash flow distribution; *(c)* cash flows following a normal distribution; *(d)* mean of the cash flow distribution being zero; *(e)* lumpy cash transfer cost and linear cash holding cost; and *(f)* negligible lead time for cash transfers. The results are given in Table 1.

The one-sample runs test for large samples[4] was conducted on the daily net cash flows of the seven branches. The null hypothesis was that, the daily net cash flows were random. The null hypothesis was not rejected at $\alpha = 0.10$. Hence, the assumption that the daily net cash flows were independent, held good for the seven branches. At the same time, this violated one of the assumptions of the Baumol model, *viz*. the existence of continuous uniform cash flows.

Mean of daily cash flows, for each month, for each branch, was tested against mean of daily cash flows for the whole year for the respective branch. The null hypothesis was that, each month's mean of daily net cash flows was equal to mean of the two years' daily net cash flows. For all branches, for most of the months, the null hypothesis was not rejected at a = 0.10. Hence, the results justified the assumption that mean of daily net cash flows was stable across months. A similar test for variance of daily net cash flows indicated that it was stable across months.

The Kolmogorov-Smirnov one sample test was conducted to verify whether the daily net cash flows followed a normal distribution. The hypothesis was rejected at $\alpha = 0.10$ for four of the seven branches. When outliers beyond $\pm\ 2\sigma$ limits were removed from the data, the hypothesis was rejected only for one branch.

A 't' test was conducted to verify the assumption that mean of the daily net cash flow distribution was zero. The hypothesis was rejected for five out of seven branches. For branches A, E and F, the hypothesis was rejected even at a = 0.001. Hence, the results were not encouraging and the possibility of drifts in the cash flow could not be rejected. Drifts imply imbalance between cash inflows and outflows and could be due to the branches being either highly deposit oriented or highly advance oriented.

The assumption of lumpy cash transfer costs was found to be valid for both physical transfers as well as for transfers through other banks using telegraphic transfer facility. The cost of physical transfer of cash consisted of out of pocket expenses for conveyance used by bank staff transporting the cash. The interest lost on idle cash or the cash holding cost was proportional to the end of the day cash balances. This was arrived at based on the average returns obtained in the previous year by way of investing excess cash in securities and fixed deposits with other banks.

The model assumed negligible lead time for cash transfers. In the present study, it was assumed that cash transfer decisions were taken only towards the end of the day. Within the day fluctuations of cash levels and within the day cash transfer decisions were not studied. Hence, lead time for cash transfers was relevant, only if cash transfers could not be achieved within the same day. The link branch and other branches structure of the particular bank was such that the physical transfers between branches and the link branch could be effected within the day itself.

Performance of the Models

Cash flow data and cost parameters of year 1 were used to arrive at the control limits of the models. The control limits were then applied to the cash flow data of year 2, and the transfer costs, opportunity costs of idle cash, total costs, etc. were estimated. The total cost consisted of sum of transfer costs and the opportunity cost of idle cash. These costs were compared with the actual costs incurred by branches in year 2 as well as with a bench mark solution obtained for year 2.

Performance of the Miller and Orr model *vis-a-vis* actual practice as well as the bench mark solution is given in Table 2.

The bench mark policies were developed through a method of exhaustive enumeration of combinations of u, z and h, where 'u' is the lower control limit, 'z' the target level and 'h' the upper control limit. Neave[5] demonstrated that when transfer costs in both directions were lumpy and equal and if the cost function was symmetric quasi convex, then the optimal policy would be a simple policy of type (u, z, h). The method of exhaustive enumeration adopted is described in the Appendix. The enumeration was done on the year 2 data to arrive at the bench mark policy.

The application of the Miller and Orr model led to lower total costs compared to actual practice. The better performance of the model could be due to the following reasons : *(a)* cash flow pattern fitted the assumptions of the model; and *(b)* the model explicitly considered trade offs between cash transfer costs and the opportunity cost of idle cash and the control system existing in the bank was probably biased towards one of the two costs, leading to lower performance.

The application of the model resulted in lower average cash balance, but increased number of cash transfers (Table 3). The transfer costs per transfer were comparatively low. Hence, the model triggered more transfers, thereby reducing the average cash balance.

If we assume that the managers were acting under a control system which sensitized them equally to both the costs, the lower number of cash transfers in actual practice could be due to the predictability of cash flows. If the branch manager predicts high outflow of cash for the early part of the following day, he would not transfer excess cash at the end of the day to the link branch.

The difference in cost between the model and the bench mark policy or the best policy obtained *post facto* ranged from 5 per cent to 70 per cent across branches. Hence, even though the performance of the Miller and Orr model was better than actual practice, it was significantly worse than that of the best solution for the given cash flow data. The lower performance of the model could be due to the following reasons: *(a)* the drift in cash flows or the mean of the cash flows being significantly different from zero; and *(b)* significant difference between the cash flow patterns of year 1 and 2, as the Miller and Orr model control limits were developed from year 1 data while the bench mark policy was developed from year 2 data on a *post facto* basis.

To check the possibility of the latter, another set of Miller and Orr model control limits was developed from the cash flow distribution of year 2 itself. The difference in total cost when the model based on year 1 data was applied, compared to the one based on year 2 data is given in Table 4. It can be seen from the Table that the cost increased for four out of seven branches. The decrease in total cost for the remaining three branches was insignificant. This clearly showed that the lower performance of the Miller and Orr model was not due to difference in the cash flow pattern between the years.

Hence, the lower performance of the Miller and Orr model could be attributed to the drift in the cash flows. This is also evident from Table 5 in which the mean of the cash flows for year 2 is tabulated against the difference in cost between the Miller and Orr model and the bench mark policy. The increase in cost for the Miller and Orr model was found to be high for branches with high mean for cash flows and low for branches with low mean for cash flows. Hence, the lower performance of the Miller and Orr model could be attributed to drift in the cash flows.

Performance of the Baumol Model

The Baumol model was recommended by one of the public sector banks for situations of drift in cash flows. Hence, it was decided to assess the performance of the Baumol model *vis-a-vis* the bench mark policy and the Miller and Orr model. The total cost figures are compared in Table 6. (Branch C was excluded as it was the only branch with negative mean cash flow).

The Baumol model fared better than the Miller and Orr model with respect to five branches. At the same time, it fared significantly worse than the bench mark policy for two branches. These were branches for which the mean cash flow was not significantly different from zero. The lower performance could also be due to the randomness in cash flows, a violation of one of the basic assumptions of the Baumol model. For branches which exhibited a drift in cash flows, the Baumol model gave good results.

Control Limits

Since, the Miller and Orr model gave unsatisfactory performance in most of the cases and the Baumol model in certain cases, a need was felt for identifying a better method for deriving the control limits. The method of exhaustive enumeration, which was used for deriving the bench mark policy had the advantages of computational and conceptual simplicity compared to other analytical models such as Markov Decision Processes. Hence, the method of enumeration given in the Appendix was used to derive control limits based on year 1 data. The control limits were then applied on year 2 data and the performance was compared with that of the bench mark policy. The results are given in Tables 7 and 8.

The difference in total cost between enumeration on year 1 cash flows and the bench mark policy ranged between 0 per cent and 6.97 per cent. The control limits and target levels were only marginally different (Table 8). Hence, control limits obtained through enumeration on previous year's cash flow were good for the year ahead. The Kolmogorov-Smirnov two sample test was conducted to check the similarity of cash flow distributions of year 1 and year 2. The hypothesis was not rejected at $\alpha = 0.10$ for five out of seven branches.

The computational time for enumeration (cpu time) in a PDP-11 system was approximately in the range of seven to eight minutes per branch. In actual practice, retention limits are determined only once in a year. Hence, this does not envisage any significant computational burden for the branches. Bank branches have increasingly started using micro processors for back office as well as front office automation. The cash flow data required for the enumeration would hence be available in floppies at the end of the year. The method of enumeration, hence, has potential for determining the control limits for day-to-day cash management.

Conclusion

Control limit models for cash management, which explicitly consider the cash related costs and cash flow distribution were tested on seven branches of a private sector scheduled commercial bank. While the Miller and Orr model and the Baumol model, recommended by some of the banks did not perform well for the branches under consideration, exhaustive enumeration on previous year's data gave good results. Enumeration being computationally simple has potential for practice.

The cash flow distributions for the two consecutive years of the study exhibited remarkable stability across the years. Further research on this aspect would throw more light on the value of control limits determined from immediately preceding year's data.

Table 1
Tests for Validity of the Assumptions of the Miller and Orr Model

Branch	Test runs	No. of Months for which the Estimate of the Parameter was not Significant: Stability of mean of cash flows	Stability of variance of cash flows	Kolomogorov Smirnov test	Test for $\mu = 0$
A	NS	17	19	NS	S
B	NS	21	16	S	NS
C	NS	18	18	S	S
D	NS	19	19	NS	NS
E	NS	20	17	NS	S
F	NS	19	21	S	S
G	NS	21	16	S	S

Notes : α = 0.10 was used for all the tests.
NS = Not Significant; and S = Significant.

Table 2
Cost Performance – Actual, Model and Bench Mark

(Total Cost in Rs.)

Branch (1)	Actual (2)	Miller & Orr model (3)	Bench mark policy (4)	Difference in total cost (5) = 3 – 4	Percentage of bench mark policy (6) = 5/4
A	14,116	4,208	2,468	1,739	70.47
B	2,797	2,679	2,420	259	10.71
C	4,972	4,538	4,312	226	5.24
D	7,985	4,430	3,681	749	20.36
E	13,069	6,115	4,612	1,503	32.58
F	5,608	4,329	3,810	519	16.41
G	3,637	2,691	2,312	379	16.41

Table 3
Cash Balance and Transfers

Branch	Actual		Miller and Orr Model	
	Average cash balance Rs. '000	Number of transfers	Average cash balance Rs. '000	Number of transfers
A	200	124	51	192
B	36	32	29	63
C	66	42	53	88
D	109	52	48	108
E	180	67	69	114
F	74	49	49	86
G	48	52	32	75

Table 4
Difference in Cost

Branch	Percentage decrease in cost over the model based on year 1 data
A	–10.56
B	0.0
C	0.89
D	2.05
E	–0.90
F	–0.27
G	0.67

Table 5
Mean Cash Flows for Year 2

Branch	Percentage increase in cost of the Miller and Orr model	Mean cash flow of year 2
A	70.47	53.65
B	10.71	0.62
C	5.24	–3.85
D	20.36	4.34
E	32.58	19.72
F	16.41	4.05
G	16.41	1.00

Table 6
Comparative Performance

(Total cost in Rs.)

Branch	*Miller and Orr model*	*Baumol model*	*Bench mark policy*	*Percentage difference in cost of Baumol model over bench mark policy*
A	4,208	2,633	2,468	6.68
B	2,679	3,141	2,420	29.88
D	4,430	3,956	3,681	7.49
E	6,115	4,637	4,612	0.54
F	4,329	4,054	3,810	6.40
G	2,691	2,669	2,312	15.43

Table 7
Method of Enumeration

(Total cost in Rs.)

Branch	*Enumeration on year 1 cash flows*	*Bench mark policy*	*Percentage increase in cost from bench mark policy*	*Kolmogorov Smirnov two sample test: year 1 versus year 2 cash flows*
A	2,469	2,468	0	S
B	2,589	2,420	6.97	NS
C	4,327	4,312	0.34	NS
D	3,695	3,681	0.38	NS
E	4,629	4,612	0.37	NS
F	3,882	3,810	1.89	S
G	2,318	2,312	0.27	NS

Notes : S = Significant at α 0.10; and NS = Not significant at $\alpha = 0.10$

Table 8
Control Limits

	Enumeration on Year 1 Cash Flows			*Bench Mark Policy*	
Branch	*Lower control limit* u	*Target level* z	*Upper control limit* h	*Target level* z	*Upper control limit* h
A	20	20	37	20	38
B	15	23	48	15	43
C	30	30	77	30	84
D	20	20	68	20	54
E	25	25	71	25	75
F	20	20	57	20	62
G	15	15	46	15	39

Appendix I
Method of Exhaustive Enumeration

The lower control limit 'u' was taken as equal to the safety stock of cash. The enumeration was done as follows :

Step 1 — 'z' and 'h' for each branch were increased from the levels given by the Miller and Orr model results. Specifically, 'z' was increased by Rs. 10,000 (10 units) and 'h' was increased by Rs. 20,000 (20 units) for each branch. $z_0 = z + 10$ and $h_0 = h + 20$. Further, $z_n = z_0$ and $h_n = h_0$.

Step 2 — Costs were computed by applying z_0 and h_0 on year 2 data. If $u = z_n = h_n$ step 7 was followed, otherwise step 3 was followed.

Step 3 — z_n was made equal to z_0 and h_n was made equal to h_0, i.e. $z_n = z_0$, $h_n = h_0$.

Step 4 — $z_n = z_n - 1$ and $h_n = h_n - 1$.

Step 5 — Costs were computed by applying z_n and h_n on year 2 data. If $z_n \neq u$, step 4 was followed, otherwise step 6 was followed.

Step 6 — $z_0 = z_0 + 1$. Step 2 was followed.

Step 7 — The least cost policy was chosen.

This way, large number of combinations of target levels and upper control limits in units of Rs. 1,000 were tried out for each branch.

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