EpiStrat: A Tool for Comparing Strategies for Tackling Urban Epidemic Outbreaks

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Abstract. Management and mitigation of epidemic outbreaks is a major challenge for health-care authorities and governments in general. In this paper, we first give a formal definition of a strategy for dealing with epidemics, especially in heterogeneous urban environments. Different strategies target different demographic classes of a city, and hence have different effects on the progression and impact of an epidemic. One has to therefore choose among various competing strategies. We show how the relative merits of these strategies can be compared against various metrics.

We demonstrate our approach by developing a tool that has an agent based discrete event simulator engine at its core. We believe that such a tool can provide a valuable what-if analysis and decision support infrastructure to urban health-care authorities for tackling epidemics. We also present a running example on an influenza-like disease on synthetic populations and demographics and compare different strategies for outbreaks.

1 Introduction

Significant progress has been made over the last century in mathematical modeling and analyses of epidemics, starting with the early work of Kermack and McKendrick [12,14] where they introduced compartmental models of epidemic and modeled epidemic progression through differential equations. Subsequent work introduced stochastic differential equations and stochastic processes [3]. The recognition that populations are not homogeneous resulted in the use of ideas from graph theory in the form of contact networks [13,15,18]. The advent of social media and the study of large graphs and complex networks resulted in a cross-breeding of ideas from both communities [1,20,21]. Ideas from physics have been applied to epidemiology as well, with good success [17,22]. Human mobility models have been incorporated to account for effects of traffic pattern on the spread of epidemics across geographies [6].

In this paper, we address a common problem faced by decision making health-care authorities during the outbreak of an epidemic in an urban setting. Large cosmopolitan cities exhibit great socio-economic and cultural diversity. While

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there are slum areas that have higher population density, lower literacy rates and incomes, there are also upmarket areas that fare better in the above parameters. In addition, there are parameters associated with individuals, like age, nutrition levels etc. that influence the progression of an epidemic.

An epidemic outbreak poses the following dilemma to urban health-care authorities and policy makers: In what proportion should different demographics be paid attention to, to minimize various effects of an epidemic outbreak?

This paper is a step towards formalization of the problem statement and development of a decision support tool based on (agent based) modeling and simulation for solving the problem. Our contribution is twofold: (i) A formal definition of a *strategy* (ii) An agent based modeling and simulation approach for comparison of various strategies in terms of economic and demographic impact. To the best of our knowledge, this is the first attempt in this direction.

We now give a brief sketch of our approach. Details can be found in later sections. We first partition the entire population into different classes. This classification can be based on criteria like geographic location or socio-economic parameters. In our running example, we use a classification based on vulnerability of individuals to the epidemic. Next, we identify a set of measures that health-care authorities intend to take, to tackle the epidemic. We work with two different types of measures: abstract measures defined in terms of susceptibility lowering and concrete measures defined in terms of tangible steps like quarantine. Finally, we formally define a strategy in terms of the proportion and schedule of applying the measures on various demographic classes obtained previously.

Through agent based simulation, we can compare different strategies against epidemic metrics like peak incidence, cumulative incidence, duration of the epidemic, economic impact and the cost associated with implementing a strategy. We develop a tool for the simulation, as well as a comparison of these metrics. The tools allows fixing of various demographic and disease related parameters. In particular, if the costs of implementing a measure on different classes are available, the tool outputs the total cost of a strategy as a byproduct. We show *cumulative* infection plots and tabulate some other metrics for illustration.

We believe that such a tool would be useful to health-care policy makers in analyzing what-if scenarios before operationalizing a strategy for tackling an epidemic.

The rest of the paper is arranged as follows. In the next section, we briefly establish the preliminaries and terminology required for the rest of the paper and discuss relevant existing literature. In Sect. 3, we discuss our approach and the tool–a description of our agent based model, strategies and costing models. In Sect. 4, we demonstrate our approach through a synthetic scenario, and compare different strategies. We conclude in Sect. 5 with a discussion of future directions.

2 Preliminaries and Previous Work

Compartmental models have been the mainstay of epidemic analysis for almost a century [5]. These models partition the population into several compartments; the number of compartments depends on the disease. A common model is the SEIR, that classifies every individual into either the "Susceptible", the "Exposed" (but not symptomatic or infectious), the "Infectious" or the "Recovered" compartment. This models holds for several common diseases like influenza, measles, Ebola virus disease etc. Indeed, we use this model for our running example.

The transition dynamics of people from one compartment to another has been modeled in different ways, including deterministic and stochastic differential equations. Deterministic models, while being simple, have the limitation that they hold for large populations, with typically homogeneous mixing assumptions among the population.

The paradigm of modeling individual agents and their behavior has been used in recent times for analysis of epidemics. While being computationally expensive, these models gained traction in epidemiology community recently because of the higher degree of accuracy that they offer [4,8,9].

A lot of work has been done in studying sociological phenomena in the context of epidemics. For example, Mao [16] and Durham and Casman [7] investigate progressive decision making process and individual response to epidemics. Funk et al. [11] model the spread of awareness during an outbreak of an epidemic and discuss how this could result in a lowering of individual susceptibility, and therefore, a smaller outbreak size. There is also work on surveillance systems and mechanisms for targeting and monitoring various interventions during outbreaks [19]. The outcome of surveillance can then be used to formulate strategies to tackle the epidemics.

As mentioned earlier, there exist several tools based on various simulation and analysis paradigms for studying epidemic progression [2,4,6]. However, what is required is a tool that allows rapid evaluation and comparison of different strategies by health-care authorities. This paper is a first step in that direction.

3 Our Approach

3.1 Agent Based Model

The Usual Scenario, When There Is No Epidemic: We give a qualitative description of the agents and the environment. The parameters and actual values set for the simulation are detailed in the next section. We simulate a town on a square grid that has two areas: a "slum" area (Area 1) which is characterized by high population density, lower education, and an upmarket area (Area 2) with sparse population and higher education. Places consists of one or more grid points and an individual is located on one grid point. There are residential areas, workplaces, schools and market places in both areas. Individuals living in the town are characterized by features that are relevant to our experiments–home address, age, occupation, work place address etc., in addition to education level. In addition to the designated work places in the upmarket area, some of the slum dwellers also work in the residential areas in the upmarket area. The unit of time that we use is one hour. Movement of individuals in the city is modeled as movement from one grid point to another.

Each individual goes about his/her routine as defined by his/her age and occupation. Working hours are between 9AM and 5PM. Workplaces and schools consist of several grid points. The movement of the employees/students within the workplace/school is modeled as a random hops between the grid points within the workplace. Office-goers visit the market place in the evening with higher probability, while home-makers visit the market place with equal probability throughout the active day. In any case, everybody returns to his/her respective home at the end of the day (8:00 PM).

Outbreak of an Epidemic: The outbreak of the (influenza-like) epidemic proceeds as follows. We begin with an initial number of infected individuals. The epidemic comes to the notice of health-care authorities only after a certain threshold number of people are infected. After a delay δ that is defined by the strategy adopted by authorities, various measures (defined later) come into effect.

Regardless of their susceptibility, a healthy individual goes about his/her routine as usual. A healthy individual is exposed to infection if there is an infected individual sharing the same grid point at the same time. The probability that a healthy individual acquires the infection is given by [4]:

$$p_i = 1 - exp(\tau \sum_{r \in R} N_r ln(1 - r s_i \rho)) \tag{1}$$

where τ is the duration of exposure, R is the set of infectivities of the infected individual at that location, N_r is the number of infectious individual with infectivity r, s_i susceptibility of individual i and ρ is the transmissibility.

After acquiring the infection, he stays asymptomatic and non-infectious for a certain duration after which he becomes infectious. When infected, every individual stays at home with some probability.

The probability that an individual recovers after i_t units of time after entering the infectious stage is given by

$$p_r = 1 - (1 - (1/r_t))^{i_t}$$

where r_t is the average recovery time.

Health-Care Response: An alert is triggered to the health-care authorities only after a certain fraction of the population gets infected. In our simulations, we assume that it is triggered when (1/25)th of the population is infected. After a delay, the authorities respond with a strategy (defined later). This delay could be due to several reasons like lack of resources or systemic inertia.

3.2 The Strategy

Classification of the population can be done in several ways. A simple classification could be simply based on the locality of the individual–all individuals of Area 1 belong to one class, while those of Area 2 belong to another class.

A more sophisticated classification is given below. In this paper, we report simulation results for this classification. As mentioned earlier, we divide the entire population of n people into three demographic classes and assign different initial susceptibilities to each of these classes.

```
if a person A satisfies at least two of the following
conditions:
    1. A.age <=10 years
    2. A.age >=70 years
    3. A.MaxEducation <= High School
    4. A.residence = Area 1
then A.Class = 0

if A satisfies all of the following conditions:
    1. 11 <= A.age <= 69
    2. A.MaxEducation >= Graduation
    3. A.residence = Area 2
then A.Class = 2

otherwise
A.Class = 1
```

Let $|C_i| = n_i$, for $i \in \{0, 1, 2\}$. It is easy to see that the above routine partitions the population into the three classes $\{C_i\}$. Thus, $n_0 + n_1 + n_2 = n$.

We report experiments based on this classification method.

A *measure* is a step taken by the health care authorities that benefits an individual (of a certain class) with some probability. In this work, we consider two qualitatively different types of measures. The first is an abstract one, defined by a lowering of susceptibility. How this lowering is brought about, is not described. For example, for our simulations, we use the following abstract measures: for an individual with natural susceptibility s, lowering to (i) $2s/3$ (ii) $s/3$ and (iii) 0 (e.g., through vaccination).

The second type involves more concrete measures like closing schools, quarantine, minimizing transmission in health centers etc. While the latter type is easy to visualize, the former serves as a comparison point, and also leaves scope for including and combining other concrete measures. For example, for our simulations, we use the following concrete measures: (i) closing of schools and a reduction of transmission in hospitals to two-third of the individual's natural susceptibility (ii) quarantine of an infected individual with some probability and (iii) vaccination of an uninfected individual with some probability.

Definition 1. Given r demographic classes, and m "measures", a *strategy* is a $r \times m$ matrix S where $S_{i,j} = (p_{i,j}, t_{i,j})$ where $p_{i,j}$ is the probability that an individual of the demographic class C_i will benefit from measure j, and this measure will be taken after a delay of $t_{i,j}$ units after epidemic alert is raised.

Essentially, a strategy defines in what proportion are different demographics targeted by the health-care authorities. The delay $t_{i,j}$ accounts for the time needed to put various measures in place. While a strategy is intuitive in the context of abstract measures, we point out that even for concrete measures, the intuition holds. For example, it is more difficult (expensive) to bring down susceptibility among individuals of class C_0 even in hospitals. Similarly, it is also more difficult to impose quarantine on such an individual. Thus, it makes sense to associate the cost of such a measure with the probability of it being used on an individual of a certain class.

In our model, a person who is subjected to a measure is eligible to be subjected to subsequent measures. For example, in the case of abstract measures, if the person's susceptibility is lowered to $2s/3$, it can further be lowered to $s/3$ and 0 later.

Definition 2. Associated with a strategy is a $r \times m$ *expense* matrix E where each entry denotes the average expense of implementing the measure in an individual of class C_i.

Thus, the total expense of a strategy is $\sum_{i=0}^{r-1} \sum_{j=0}^{m-1} n_{i,j} E_{i,j}$, where $n_{i,j}$ is the number of individuals of class C_i who were subjected to measure j. Our partitioning of the population also allows to provide a rough estimate of the economic impact of the epidemic. For that, we associate an average economic value v_i with each individual of a demographic class C_i. Thus if l_i individuals of C_i get infected in a epidemic, we say that the economic impact of the epidemic is $\sum_i l_i v_i$.

Thus, different strategies yield different progressions of the epidemic. It results in different shapes of the epidemic curve, different economic impacts, and finally, different expenses.

3.3 The Tool

The tool is implemented in python 2.7. The simulator reads a configuration file that contains all necessary parameters to set up the environment, agents and disease specific parameters. The source code, instruction manual and examples are available at https://github.com/radh3110/EpiDemoSim-Project.

4 Simulation Results

To run the agent based simulation, we need to fix the properties of the disease, demographic and geographic settings of the model, and behavioral properties of the agents.

As mentioned earlier, we assume an influenza-like epidemic, and hence use values reported in literature for [10]. These are shown in Table 1. We emphasize that since populations are synthetic and so are some infection characteristics, the simulations results presented do not relate to any specific real life epidemic example. The examples are purely to demonstrate our approach and tool. Table 2 shows various parameters set in the agent model.

Recall that the classification scheme that we use in this paper results in three classes C_0, C_1 and C_2 of n_0, n_1 and n_2 individuals respectively, with decreasing

Table 1. Epidemic parameters

Transmissibility (ρ in Eq. 1)	0.029
Infectivity range (R in Eq. 1)	0.1–0.9
Expected duration of latent period	26 h
Incubation period duration	30–72 h
Recovery time	60 h

Table 2. Model parameters

People in Area 1	4000
People in Area 2	2000
Initially Infected	50
Probability of going to the market between 9AM to 5PM	0.05
Probability of going to the market between 6PM to 8PM	0.2
Probability of going to a hospital when symptomatic	0.4
Education Levels	0 to 4

natural susceptibilities. For the current simulation, we have $n_0 = 4029$, $n_1 = 1348$ and $n_2 = 623$. We assign a natural susceptibility of 0.8 to individuals of C_0, 0.5 to those of C_1 and 0.3 to those of C_2.

4.1 Measures

We begin by defining strategies that are agnostic to demographic classification. Any measure among the three (see Table 3) is implemented across all communities after the same delay. The delay depends on the measure–100, 80 and 50 time units for measures m_0, m_1 and m_2 respectively. We will first describe abstract strategies and then concrete strategies over which we report our simulations.

S_Φ is the "strategy" when there is no intervention on the part of health-care authorities. Therefore, susceptibility remain the same as their natural susceptibility for all individuals of all demographic classes. On the other hand, S_{m_0} is the strategy where we vaccinate every individual in the population. Finally, S_{red} is the strategy when the authorities do not intervene, but the susceptibilities of all individuals is one-third of the natural susceptibility associated to their respective class. This is to depict the situation when the general health-care, civic and educational infrastructure is so good that the susceptibilities are lower to begin with. Note that these strategies have the same interpretation in both abstract and concrete settings.

The abstract strategies S_{m_2} and S_{m_1} are strategies where the attempt is to lower individual susceptibilities to two-third and one-third respectively for all individuals of all classes.

The concrete strategy S_{m_2} is one where the susceptibilities of all people when in hospitals falls to $2s/3$ with probability 1 and schools close. The concrete strategy S_{m_1} is one where every symptomatic person is quarantined with probability 1.

Table 3. Common strategies

	m_0	m_1	m_2	m_Φ
S_Φ (for all classes)	0	0	0	1
S_{m_2} (for all classes)	0	0	1, 50	0
S_{m_1} (for all classes)	0	1, 80	0	0
S_{m_0} (for all classes)	1, 100	0	0	0

S_{red}: No intervention, susceptibility 1/3 of respective natural susceptibilities of all classes.

Table 4 shows more complex strategies that target communities preferentially (and also, an example cost matrix). For example, the strategy in Table 4(a) targets community C_0. We explain the first row of this table, and leave the rest to the reader.

Table 4. Targeting specific demographics

(a) S_1: Focus on C_0.

	m_0	m_1	m_2
C_0	2/5, 100	1/5, 80	1/5, 50
C_1	1/5, 100	1/5, 80	2/5, 50
C_2	1/5, 100	1/5, 80	2/5. 50

(b) S_2: Focus on C_1.

	m_0	m_1	m_2
C_0	1/5, 100	1/5, 80	2/5, 50
C_1	2/5, 100	1/5, 80	1/5, 50
C_2	1/5, 100	1/5, 80	2/5, 50

(c) S_3: Focus on C_2

	m_0	m_1	m_2
C_0	1/5, 100	1/5, 80	2/5, 50
C_1	1/5, 100	1/5, 80	2/5, 50
C_2	2/5, 100	1/5, 80	1/5, 50

(d) The cost matrix. These numbers are synthetic and chosen arbitrarily for purpose of illustration.

	m_0	m_1	m_2
C_0	49	36	25
C_1	125	48	27
C_2	81	16	1

The abstract strategy reduces the susceptibility of a person of community C_0 to two-thirds with probability 1/5 after a delay of 50 time units, to one-thirds with probability 1/5 after a delay of 80 time units and gets vaccinated with probability 2/5 after a delay of 100 time units.

On the other hand, the corresponding concrete strategy reduces the susceptibility of a person of community C_0 to two-thirds with probability 1/5 after a delay of 50 time units, when in a hospital[1]. This concrete strategy also quarantines a symptomatic person of C_0 with probability 1/5 after a delay of 80 time units and finally, vaccinates with a probability of 2/5 after a delay of 100 time units.

In our simulations, we keep the probability of a measure being administered on a person independent of the previous measures administered on him/her. For example, the same person can get susceptibility lowered to s/3 and the subsequently get vaccinated. This assumption need not be true in general and can be relaxed. For example, in concrete strategies, it does not make sense for

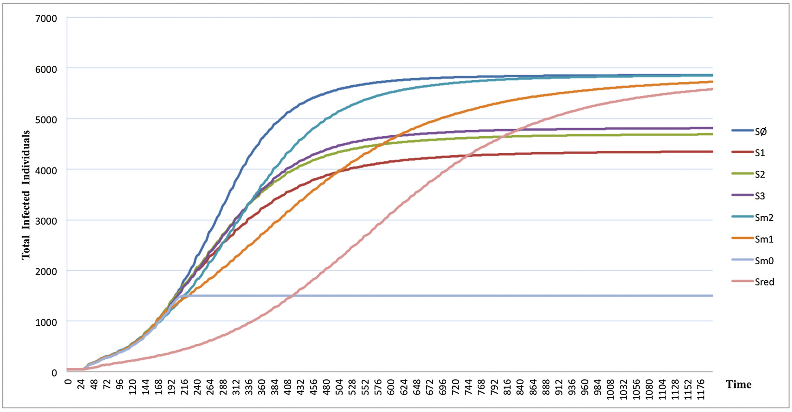

Fig. 1. Epidemic curves for abstract measures

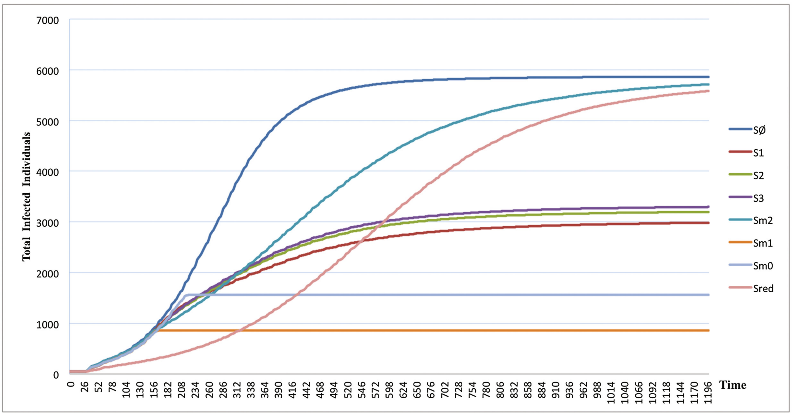

Fig. 2. Epidemic curves for concrete measures. Curves for S_{Φ} and S_{red} are reproduced here as well for easy comparison.

[1] Additionally, we close down all schools in our simulations.

Table 5. Auxiliary data generated for simulations with abstract measures

Strategy	Infected	Peak	Peak time	Alert triggered (hrs)	C_0 Infected	C_1 infected	C_2 Infected	C_0 Vaccinated	C_1 Vaccinated	C_2 Vaccinated	Total Vaccines	Economic impact	Total cost
S_{Φ}	5862	46	297	–	3905	1288	617	0	0	0	0	8336	0
S_1	4349	33	206	76	2726	1063	508	1609	273	120	2002	6381	250295.5
S_2	4690	36	250	72	3289	873	507	827	527	123	1477	6528	292334.9
S_3	4812	37	264	75	3290	1072	399	805	265	249	1319	6633	260614
S_{m_2}	5853	36	293	69	3904	1285	612	0	0	0	0	8313	137744.0
S_{m_1}	5730	28	194	72	3861	1253	565	0	0	0	0	8065	241284.0
S_{m_0}	1502	32	190	77	1099	265	88	4029	1348	623	6000	1895	416384.
S_{red}	5586	25	556	–	3809	1194	532	0	0	0	0	7795	0

Table 6. Auxiliary data generated for simulations with concrete measures

Strategy	Infected	Peak	Peak time	Alert triggered (hrs)	C_0 Infected	C_1 infected	C_2 Infected	C_0 Vaccinated	C_1 Vaccinated	C_2 Vaccinated	Total vaccines	Economic impact	Total cost
S_1	2984	24	149	79	1890	718	326	1546	204	107	1857	4304	314077.5
S_2	3198	25	159	80	2243	586	318	764	479	111	1354	4371	354179.3
S_3	3295	24	183	84	2247	729	268	789	254	210	1253	4512	348965
S_{m_2}	5714	25	256	80	3876	1234	604	0	0	0	0	8160	278654.0
S_{m_1}	855	23	143	83	631	134	50	0	0	0	0	1019	425647.0
S_{m_0}	1559	34	199	84	1169	261	78	4029	1348	623	6000	1927	387781

the same person to be quarantined and later vaccinated, especially in SEIR epidemics.

We also associate an economic impact of $i + 1$ if an individual of class C_i gets sick. Figure 1 shows *cumulative* infection curves for abstract measures. It can be seen that but for universal vaccination, targeted strategies perform better than classification agnostic strategies. Table 5 shows various other auxiliary data generated, including how many people of each class get infected.

Figure 2 shows *cumulative* infection curves for concrete measures and Table 6 the corresponding auxiliary data. While targeted strategies outperform classification agnostic strategies here as well, the interesting observation here is that quarantine (with some probability, after 80 time units) is a better measure than vaccination (with some probability, after 100 time units).

It can also be seen that the infections are always less in the class targeted by a strategy.

5 Conclusion and Future Work

In this paper we introduced a formal definition of strategy for handling epidemics, and report a tool that allows evaluation of different strategies. This tool, we believe, will be very useful for performing what-if analyses during outbreaks. However, for the most part, the simulations that we report are over synthetic data. An immediate future goal is to incorporate real data into the model.

There are several features that can be added to the tool itself. One direction is to increase the set of strategies and measures. Another direction would be to allow a lot of configuration parameters to be specified by the user through a simple interface. This would include strategy and measure specification in addition specification of agent, environment and disease parameters.

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